HW6 – Due Wednesday, November 30, start of class

No electronic submissions, only hard copy

1. (10 points) Let (X, d) and (Y, ρ) be metric spaces, $f : X \to X$ and $g : Y \to Y$ are continuous and onto. Now assume there is an $\alpha : X \to Y$ which is bijective and bi-continuous and $\alpha \circ f = g \circ \alpha$ or as a diagram

$$\begin{array}{ccc} X & \xrightarrow{f} & X \\ \Big\downarrow \alpha & & \Big\downarrow \alpha \\ Y & \xrightarrow{g} & Y \end{array}$$

(This situation is described by saying that (X, f) is *conjugate* to (Y, f) by α.)

Show the following:

(a) If x is a k-periodic under f, then $\alpha(x)$ is a k-periodic under g.

(b) Let P be the set of periodic points under f and P' be the set of periodic points under g. If P is dense in X, then P' is dense in Y.

(c) If $o(x, f)$ is dense in X, then $o(\alpha(x), g)$ is dense in Y

(d) Now assume that α is bi-Lipschitz. If f is expansive, then g is expansive.

(e) A point x is called f-recurrent if there is a sequence of integers $n_i \to \infty$ with $f^{n_i}(x) \to x$. If x is f recurrent, show that $\alpha(x)$ is g-recurrent.

2. (10 points) Let $Z \subset \Sigma_2^+$ be the set of all sequences in Σ_2^+ so that whenever $s_i = 0$ then $s_{i+1} = 1$. In other words, Z consists of only those sequences in Σ_2^+ which never have two consecutive zeros.

(a) Show that Z is closed.

(b) Show that if $\underline{s} \in Z$, then $\sigma(\underline{s}) \in Z$.

(c) Let P be the set of periodic points in Z under σ. Show that P is dense in Z.

(d) Show that there a sequence $\underline{t} \in Z$ so that $o(\underline{t}, \sigma)$ is dense in Z.

(e) How many fixed points does σ^3 have in Z?.