‘ HW 6 • SPRING 2020 • PROF. BOYLAND

1. Let
$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$$
Compute (by hand) the thin or reduced SVD of A. So find matrices with $A = U\Sigma V^T$ with U a 3×2 matrix with orthonormal columns, Σ a 2×2 diagonal matrix and V a 2×2 orthogonal matrix. Now write A as
$$A = \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T$$

2. Your answer must include your code and the results of running it and any figures. You should use built-in or library functions of your system. You will need to be able to import a gray scale image and turn it into a $m \times n$ matrix of floating point numbers and manipulate this and then display the result. One example in Matlab is below (there are other built-in images) and if you are using another platform you can import your own image.

```
bb = imread('board.tif'); %reads the built-in image file
bb = double(bb); %Changes it to a floating point matrix
A = .3 * bb(:, :, 1) + .6* bb(:,:,2) + .1*bb(:,:,3); %weighted average
% of three color channels to get grayscale
figure; imshow(A, []); % displays  the image
```

So A is the $m \times n$ matrix, its entries are floating point numbers between 0 and 255. Let $k = \min\{m, n\}$.

(a) Load an image file and transform it into a floating point matrix A

(b) Compute the SVD $A = U\Sigma V^T$.

(c) Plot i vs σ_i with $i = 1, \ldots, k$

(d) Let
$$A_i = \sum_{j=1}^{i} \sigma_j \vec{u}_j \vec{v}_J^T.$$
In class we showed that $\|A_i - A\|_2 = \sigma_{i+1}$. Verify this by computing the max over $i = 1, \ldots, k-1$ of $|\|A_i - A\|_2 - \sigma_{i+1}|$.

(e) By displaying A and then A_i for various i, estimate the smallest i for which A_i provides a reasonably accurate verion of the full resolution image given by A. Include a picture of the image for that i in your solutions.