

$$\textcircled{1} A^T A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad \lambda = 3, 1 \quad \text{so } \sigma_1 = \sqrt{3} \\ \sigma_2 = 1$$

$$\lambda = 3, \text{ e.v.} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{so normalized } \vec{v}_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$\lambda = 1, \text{ e.v.} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{so normalized } \vec{v}_2 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

$$\vec{u}_1 = \frac{A\vec{v}_1}{\sigma_1} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1/\sqrt{2} \\ 2/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$$

$$\vec{u}_2 = \frac{A\vec{v}_2}{\sigma_2} = 1 \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}$$

$$\text{so } U = \begin{bmatrix} 1/\sqrt{6} & 1/\sqrt{2} \\ 2/\sqrt{6} & 0 \\ 1/\sqrt{6} & -1/\sqrt{2} \end{bmatrix} \quad \Sigma = \begin{bmatrix} \sqrt{3} & 0 \\ 0 & 1 \end{bmatrix} \quad V = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$\sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T = \sqrt{3} \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} + 1 \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 1/2 \\ 1 & 1 \\ 1/2 & 1/2 \end{bmatrix} + \begin{bmatrix} 1/2 & -1/2 \\ 0 & 0 \\ -1/2 & 1/2 \end{bmatrix}$$