HW 8 • FALL 2019 • PROF. BOYLAND

The Fourier transform of a function $f : \mathbb{R} \to \mathbb{R}$ is given by

$$\hat{f}(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-ist}\, dt.$$

1. If f is real-valued, show that $\hat{f}(-s) = \overline{\hat{f}(s)}$.
2. Show that $\mathcal{F}(tf(t))(s) = i\frac{d\hat{f}(s)}{ds}$. Hint: write down $i\hat{f}(s)$ using the definition above, take the derivative and then move the derivative inside the integral.
3. Using the various properties we derived in class, express the Fourier transform of

$$\frac{d(f(at-b))}{dt}$$

in terms of $\hat{f}$.

4. We showed in class that if $f(t) = e^{-t^2/2}$, then $\hat{f}(s) = e^{-s^2/2}$. Use this fact along with the properties we derived in class to compute the Fourier transforms of the following functions.
   (a) $f(t) = e^{-7t^2}$.
   (b) $f(t) = e^{-(3t+2)^2/2}$.
   (c) $f(t) = 3te^{-t^2/2}$.