1. Let A be a full rank $(m \times n)$-matrix with $m \geq n$. Recall that its pseudoinverse is $A^+ = (A^T A)^{-1} A^T$.
   (a) Recall that the thin SVD for A is $A = \hat{U}\hat{\Sigma}V^T$ where $\hat{U}$ is $(m \times n)$ with orthonormal columns, $\hat{\Sigma} = \mathrm{diag}(\sigma_1, \ldots, \sigma_n)$, and V is $(n \times n)$ and orthogonal. Show that $A^+ = V\hat{\Sigma}^{-1}\hat{U}^T$ where $\hat{\Sigma}^{-1} = \mathrm{diag}(\sigma_1^{-1}, \ldots, \sigma_n^{-1})$
   (b) Recall that the thin R-decomposition for A is $A = \hat{Q}\hat{R}$ where $\hat{Q}$ is $(m \times n)$ with orthonormal columns and $\hat{R}$ is $(n \times n)$ and upper triangular. Show that $A^+ = \hat{R}^{-1}\hat{Q}^T$.
2. Let
$$A = \begin{pmatrix} 3 & 5 \\ 4 & -5 \end{pmatrix}.$$
   Compute (by hand) the QR-decomposition of A. (Since A is square its thin and regular decompositions are the same.)
3. Your answer must include your code and the results of running it. You should use built-in or library functions of your system. You need to be careful that your system is computing the thin QR or SVD, or else you will have to change the formulas you use. In Matlab: `qr(A, 0)` and `svd(A, 'econ')`.
   (a) Build the (10×5)-matrix A given by $A_{i,j} = \frac{1}{\sin(i+j-1)}$ and the (10×1) vector $\vec{b}$ given by $b_i = i$.
   (b) Compute the SVD of A and use the results of 1(a) to compute the least squares solution to $A\vec{x} = \vec{b}$.
   (c) Compute the QR decomposition of A and use the results of 1(b) to compute the least squares solution to $A\vec{x} = \vec{b}$.