For a function $f$ in $L^1([−\pi, \pi]^2)$ its two-dimensional Fourier series is given by

$$f(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} F_{m,n} \frac{e^{i(mx+ny)}}{2\pi}$$

where

$$F_{m,n} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y) \frac{e^{-i(mx+ny)}}{2\pi} \, dx \, dy$$

For a function $f$ in $L^1(\mathbb{R}^2)$ it two-dimensional Fourier transform is given by

$$\hat{f}(r, s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-i(rx+sy)} \, dx \, dy$$

1. Let $f(x, y) = 1$ when $-\pi/2 \leq x \leq \pi/2$ and $-\pi/2 \leq y \leq \pi/2$ and $f(x, y) = 0$ for all other $(x, y)$.
   (a) Find the two-dimensional Fourier Series of $f$.
   (b) Find the two-dimensional Fourier transform of $f$.

2. Let $f(x, y) = xy$ when $-\pi \leq x \leq \pi$ and $-\pi \leq y \leq \pi$ and $f(x, y) = 0$ for all other $(x, y)$.
   (a) Find the two-dimensional Fourier Series of $f$.
   (b) Find the two-dimensional Fourier transform of $f$.

3. Let $\phi(x, y) = \cos(2x + 3y)$.
   (a) Draw three level sets where $\phi(x, y) = 1$ and three level sets where $\phi(x, y) = -1$.
   (b) The wave length of $\phi$ is the distance between two adjacent level sets of the same value, for example, between two adjacent level sets with $\phi(x, y) = 1$. What is the wave length of $\phi$?