HW 9 • FALL 2019 • PROF. BOYLAND

For a function f in $L^1([-\pi,\pi]^2)$ its two-dimensional *Fourier series* is given by

$$f(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} F_{m,n} \, \frac{e^{i(mx+ny)}}{2\pi}$$

where

$$F_{m,n} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x,y) \frac{e^{-i(mx+ny)}}{2\pi} \, dx \, dy$$

For a function f in $L^1(\mathbb{R}^2)$ it two-dimensional *Fourier transform* is given by

$$\hat{f}(r,s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-i(rx+sy)} \, dx \, dy$$

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1. Let $f(x,y) = 1$ when $-\pi/2 \leq x \leq \pi/2$ and $-\pi/2 \leq y \leq \pi/2$ and $f(x,y) = 0$ for all other (x,y).
   (a) Find the two-dimensional Fourier Series of f.
   (b) Find the two-dimensional Fourier transform of f.
2. Let $f(x,y) = xy$ when $-\pi \leq x \leq \pi$ and $-\pi \leq y \leq \pi$ and $f(x,y) = 0$ for all other (x,y).
   (a) Find the two-dimensional Fourier Series of f.
   (b) Find the two-dimensional Fourier transform of f.
3. Let $\phi(x,y) = \cos(2x + 3y)$.
   (a) Draw three level sets where $\phi(x,y) = 1$ and three level sets where $\phi(x,y) = -1$.
   (b) The *wave length* of ϕ is the distance betwen two adjacent level sets of the same value, for example, between two adjacent level sets with $\phi(x,y) = 1$. What is the wave length of ϕ?