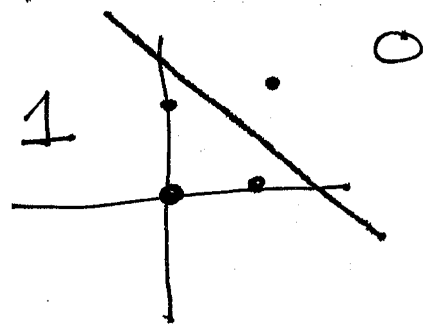
#04soln

① (a) $F(x_1, x_2) = \sigma(w_1 x_1 + w_2 x_2 + b) = 1$ and is $= 0$

when $w_1 x_1 + w_2 x_2 + b \geq 0$

when $w_1 x_1 + w_2 x_2 + b < 0$

The easiest method is to plot the points and find a decision line.



There are many such but one contains the points $(\frac{3}{2}, 0)$ and $(0, 3/2)$ and so has equation

$2x_1 + 2x_2 - 3 = 0$. Now pluggin in $(0,0)$ we get $-3 < 0$ but we need > 0 so we switch signs so $w_1 = -2$, $w_2 = -2$, $b = 3$

(b) we require

$\sigma(w_1 \cdot 0 + w_2 \cdot 0 + b) = 1$

$\sigma(w_1 1 + w_2 \cdot 0 + b) = 1$

$\sigma(w_1 \cdot 0 + w_2 \cdot 1 + b) = 1$

From the first equation and def of the ramp $b = 1$

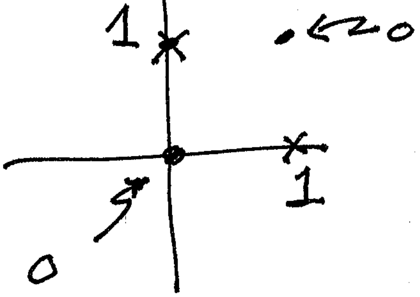
then the last two eq are $w_1 + b = w_1 + 1 = 1$

and so $w_1 = 0$, similarly $w_2 = 0$

but then for 1,1 we get $\sigma(0 \cdot 1 + 0 \cdot 1 + 1) = 1$

but that needs to be zero, so impossible.

2a



There is no decision line (easy to prove), and so no single neuron net

(2b) $F(x_1, x_2) = 1 \cdot \sigma_R(x_1 + x_2) - 2\,\sigma_R(x_1 + x_2 - 1)$

computing

$$F(0,0) = \sigma_R(0) - 2\,\sigma_R(-1) = 0 - 0 = 0$$
$$F(1,1) = \sigma_R(2) - 2\,\sigma_R(1) = 2 - 2 = 0$$
$$F(1,0) = \sigma_R(1) - 2\,\sigma_R(0) = 1 - 0 = 1$$
$$F(0,1) = \sigma_R(1) - 2\,\sigma_R(0) = 1$$