

#1 (a) we previously computed

$$\chi_{\frac{\pi}{2}}(z) = \frac{1}{2} + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{\sin(n\pi/2)}{n\pi} e^{inz}$$

Now $f(x,y) = \chi_{\frac{\pi}{2}}(x) \chi_{\frac{\pi}{2}}(y)$ and so

$$f(x,y) \sim \left(\frac{1}{2} \cdot \frac{1}{2}\right) + \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} \frac{1}{2} \frac{\sin m\pi/2}{m\pi} e^{imx}$$

$$+ \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{1}{2} \frac{\sin n\pi/2}{n\pi} e^{iny} + \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{\sin m\pi/2 \sin n\pi/2}{m n \pi^2} e^{i(mx+ny)}$$

(b) we also computed

$$\hat{\chi}_{\frac{\pi}{2}}(s) = \frac{1}{\sqrt{2}\pi} \frac{2 \sin \pi/2 s}{s}$$

$$\begin{aligned} \text{So } \hat{f}(r,s) &= \frac{2}{\sqrt{2}\pi} \frac{\sin \pi/2 r}{r} \cdot \frac{2}{\sqrt{2}\pi} \frac{\sin \pi/2 s}{s} \\ &= \frac{2}{\pi} \frac{\sin(\pi/2 r) \cdot \sin(\pi/2 s)}{rs} \end{aligned}$$

(#2a) we previously computed when $g(x) = x$ on $[-\pi, \pi]$

Ans b

$$g(z) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^n i}{n} \quad (g_0 = 0)$$

Since $f(x, y) = g(x)g(y)$.

$$f(x, y) = \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^m i}{m} \frac{(-1)^n i}{n} e^{i(mx+ny)}$$

$$= \sum_{m \neq 0} \sum_{n \neq 0} \frac{(-1)^{m+n}}{mn} e^{i(mx+ny)}.$$

(#2b) we also computed

$$\hat{g}(s) = \frac{2i}{\sqrt{2\pi}} \left[\frac{\pi \cos \pi s}{s} - \frac{\sin \pi s}{s^2} \right]$$

so

$$\hat{f}(r, s) = \hat{g}(r) \hat{g}(s)$$

$$= -\frac{4}{2\pi} \left(\frac{\pi \cos \pi r}{r} - \frac{\sin \pi r}{r^2} \right) \left(\frac{\pi \cos \pi s}{s} - \frac{\sin \pi s}{s^2} \right)$$

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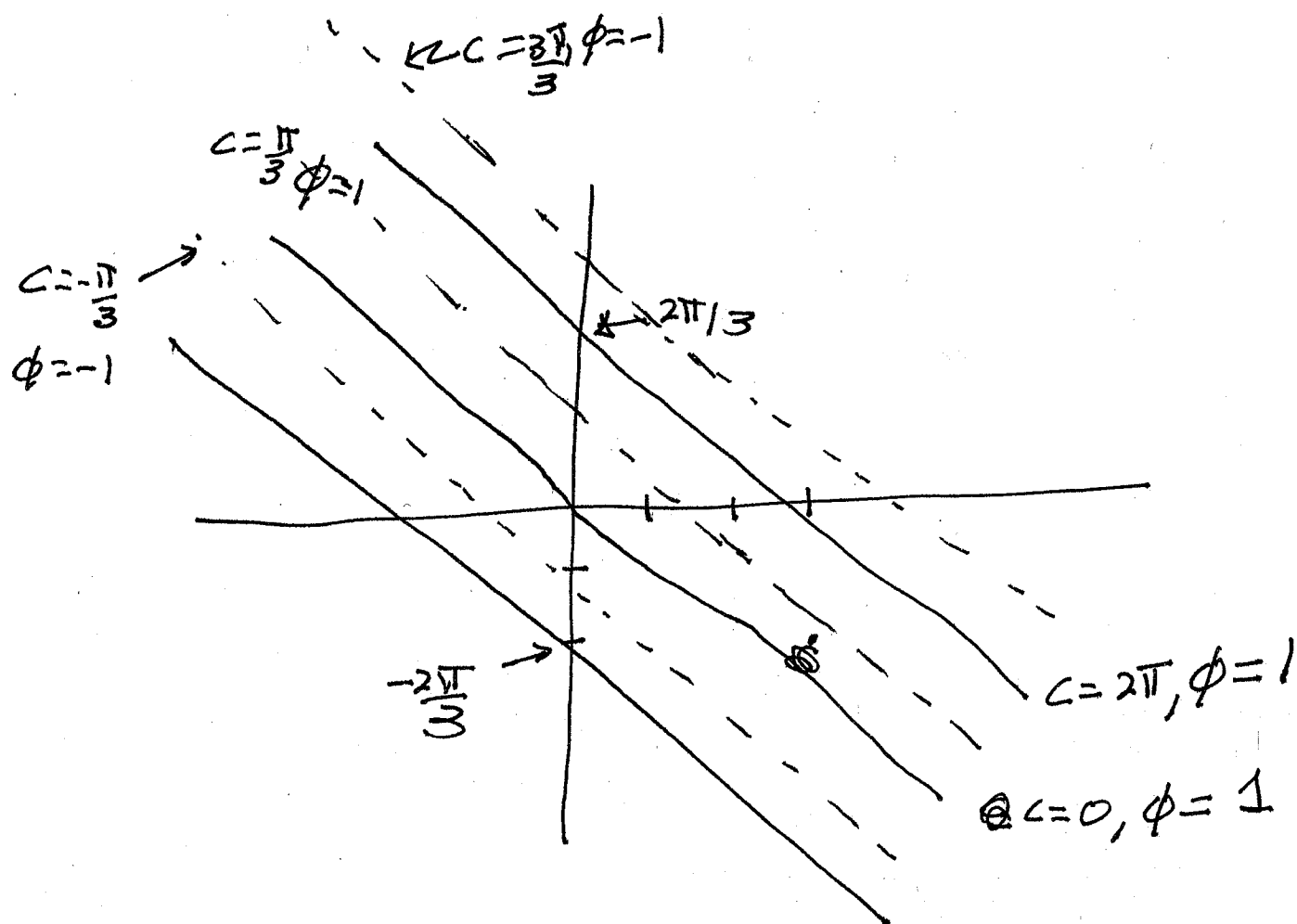
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$$2x + 3y = 0$$

$$2x + 3y = C \Rightarrow 3y = -2x + C$$

$$y = -\frac{2}{3}x + \frac{C}{3}$$



$$\text{wave length} = \frac{2\pi}{\sqrt{4+9}} = \frac{2\pi}{\sqrt{13}}$$