MATHEMATICS AND THE LIFE SCIENCES

Department of Mathematics
University of Florida

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Reinhard Laubenbacher
Laboratory for Systems Medicine
Department of Medicine
University of Florida
Gainesville, FL

reinhard.laubenbacher@medicine.ufl.edu

UF UNIVERSITY of FLORIDA

https://physicstoday.scitation.org/do/10.1063/PT.6.1.20201006a/full/

Mathematics ⟷ Life Sciences

‘Modularity is a widespread property in biological systems.’

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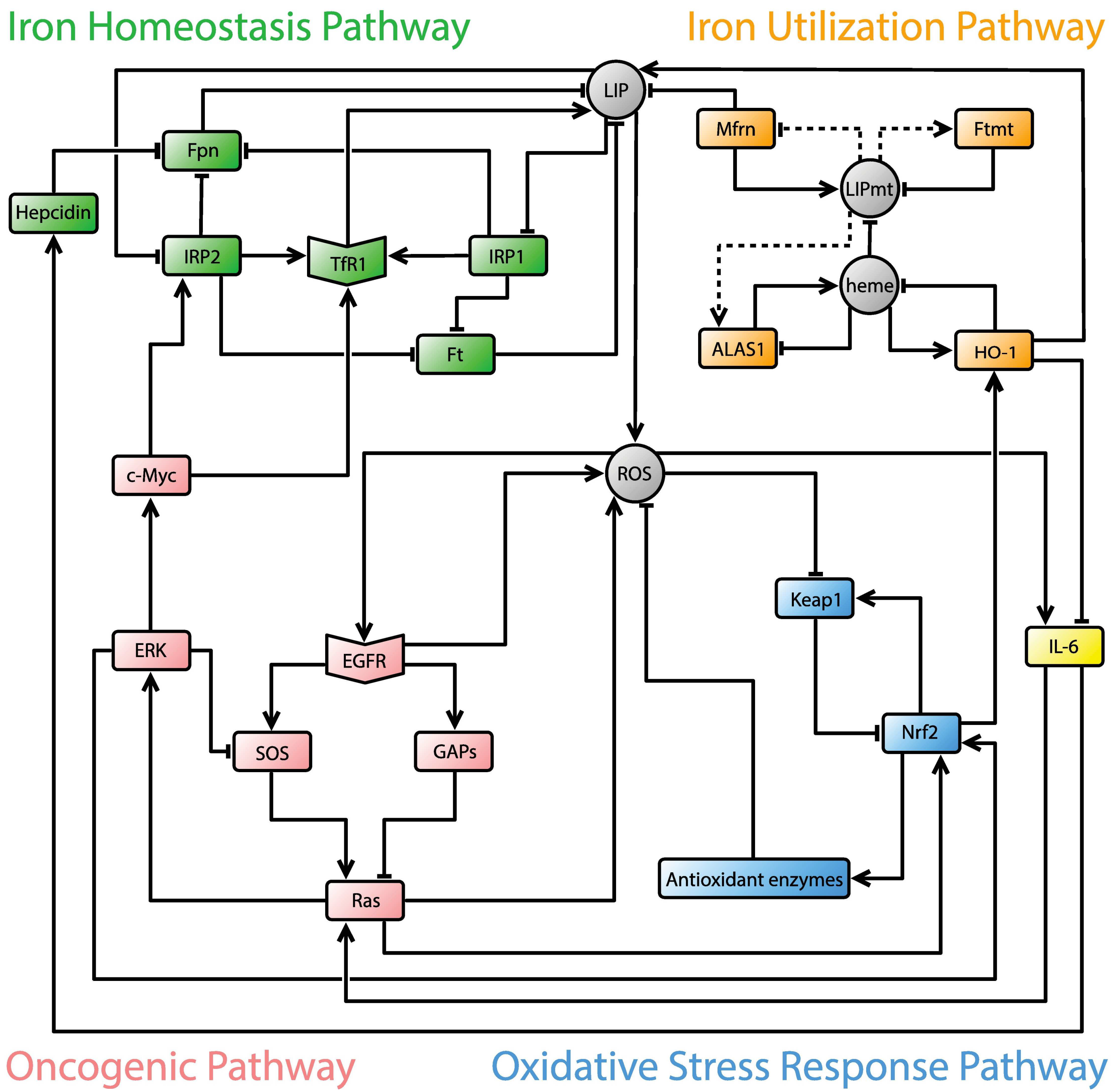
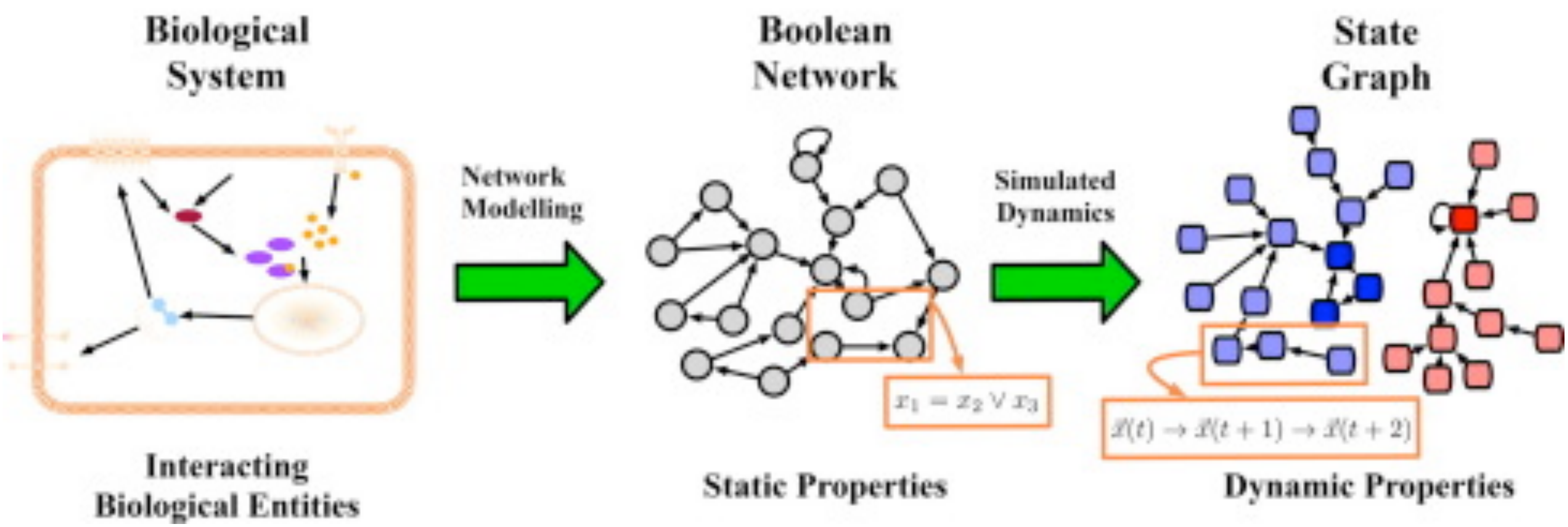
How is this property reflected in mathematical models of these systems?



Chifman et al., PLOS Comp. Biol., 2017



Schwab et al., Comp. Struct. Biotech. J., 2020

A modularity theory for dynamical systems

- What are the characteristics of models that describe molecular systems?
- What is the mathematical analog of a biological module, and how can one describe its dynamics?
- How can one decompose a given system?
- How can one parameterize all the systems one can construct from a given set of modules?

Bulletin of Mathematical Biology (2010)
DOI 10.1007/s11538-010-9501-z

ORIGINAL ARTICLE

The Dynamics of Conjunctive and Disjunctive Boolean Network Models

Abdul Salam Jarrah[a,b,*], Reinhard Laubenbacher[a], Alan Veliz-Cuba[a]

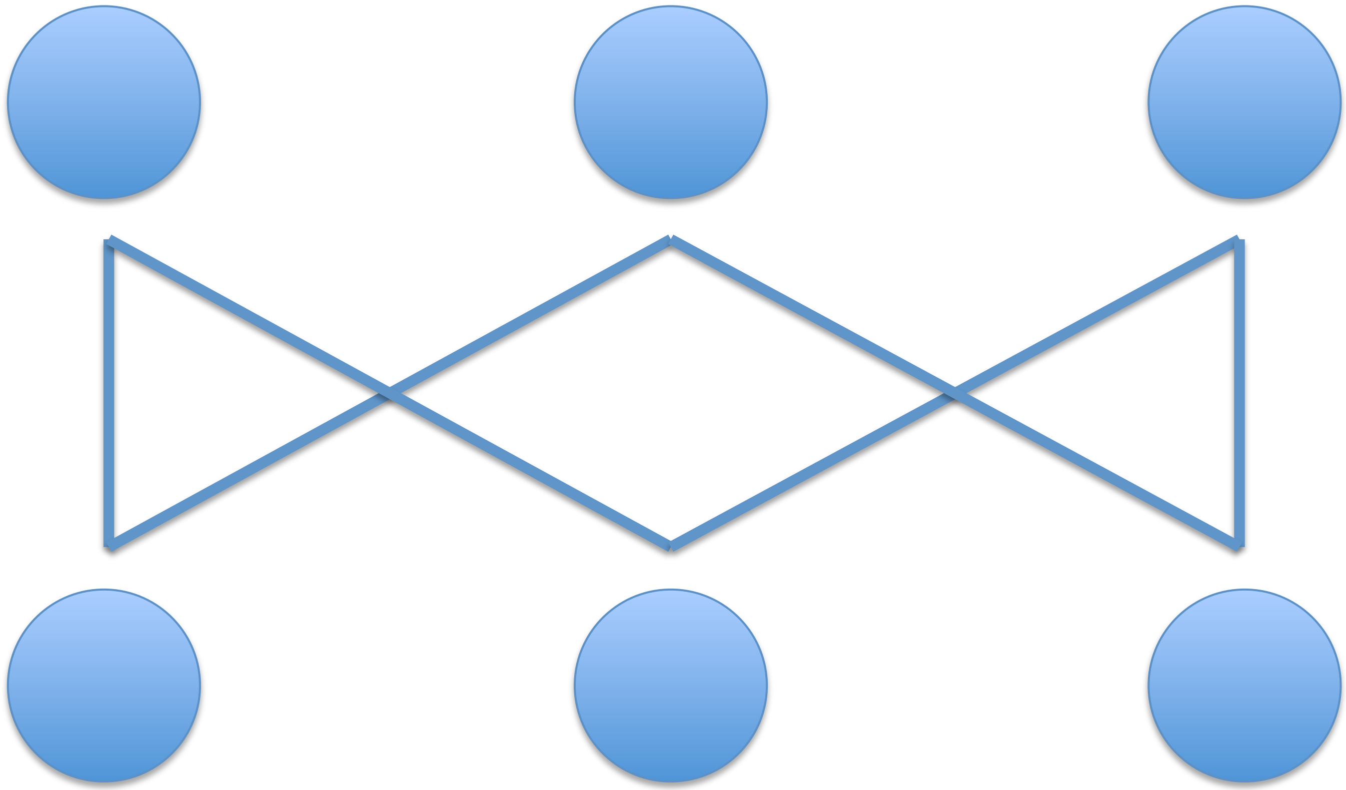
[a] *Virginia Bioinformatics Institute, Virginia Tech, Blacksburg, VA 24061-0477, USA*
[b] *Present address: Department of Mathematics and Statistics, American University of Sharjah, Sharjah, UAE*

The number of periodic points

Theorem 3 *Let f be a conjunctive Boolean network whose dependency graph is strongly connected and has loop number c. If $c = 1$, then f has the two fixed points $(0, 0, \ldots, 0)$ and $(1, 1, \ldots, 1)$ and no other limit cycles of any length. If $c > 1$ and m is a divisor of c, then the number of periodic states of period m is*

$$|A(m)| = \sum_{i_1=0}^{1} \cdots \sum_{i_r=0}^{1} (-1)^{i_1+i_2+\cdots+i_r} 2^{p_1^{k_1-i_1} p_2^{k_2-i_2} \cdots p_r^{k_r-i_r}},$$

where $m = \prod_{i=1}^{r} p_i^{k_i}$ is the prime factorization of m, that is $p_1, \ldots, p_r$ are distinct primes and $k_i \geq 1$ for all i.



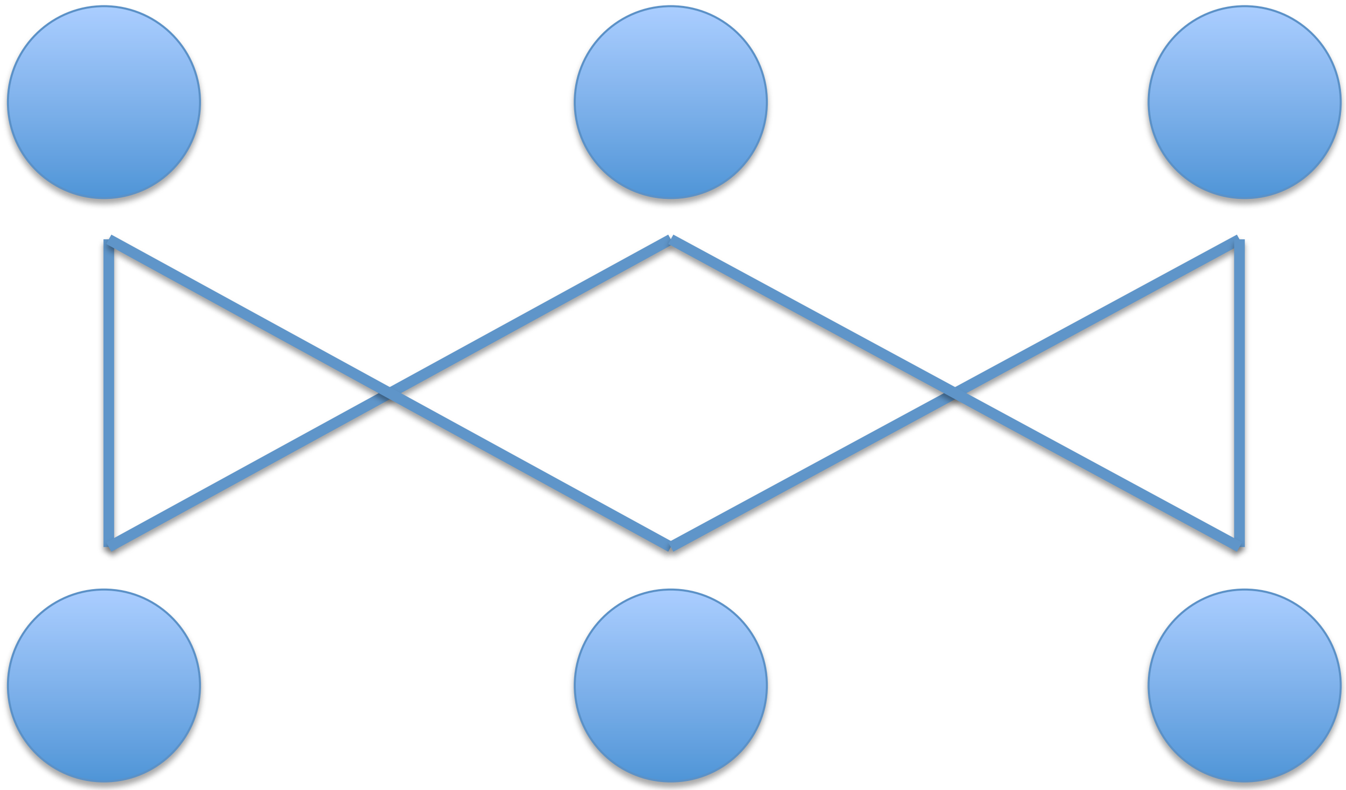
Theorem 6.2. *Consider the function*

$$\mathcal{L}(z_1,\ldots,z_t) := \sum_{\mathcal{J}\subseteq\Omega} (-1)^{|\mathcal{J}|+1} \prod_{j\in\bigcap_{J\in\mathcal{J}} J} z_i\,.$$

Then for any conjunctive Boolean network f with subnetworks $h_1,\ldots,h_t$ and Ω its set of maximal antichains in the poset of f, we have

$$\mathcal{L}\big(\mathcal{C}(h_1),\ldots,\mathcal{C}(h_t)\big) \leq \mathcal{C}(f). \tag{9}$$

Here, the function $\mathcal{L}$ is evaluated using the "multiplication" described in Corollary 3.5. This inequality provides a sharp lower bound on the number of limit cycles of f of a given length.



“Conjecture:” For any conjunctive Boolean network, there is a filtration by subnetworks such that successive “quotient networks” are disjoint unions of strongly connected networks.

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Jordan-Hölder Theorem: Every finite group has a filtration by normal subgroups, such that successive quotient groups are simple.

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Jordan-Hölder Theorem: Every finite group has a filtration by normal subgroups, such that successive quotient groups are simple.

Hölder Program: Classify all simple groups and parameterize all extensions of a group by a simple group. This provides a classification of all finite groups.

A Hölder Program For Boolean Networks:

- Determine a class of BNs that is "sufficiently rich."
- Determine an analog of simple groups and their dynamics.
- Prove an analog of the JH Theorem.
- Parameterize extensions of BNs.

(And show relevance to the biological concept.)

A meta-analysis of Boolean network models reveals design principles of gene regulatory networks

Claus Kadelka[a,*], Taras-Michael Butrie[b,1], Evan Hilton[c,1], Jack Kinseth[a,1], Haris Serdarevic[a,1]

[a]Department of Mathematics, Iowa State University, Ames, Iowa 50011
[b]Department of Aerospace Engineering, Iowa State University, Ames, Iowa 50011
[c]Department of Computer Science, Iowa State University, Ames, Iowa 50011

arXiv:2009.01216v1 [q-bio.MN] 2 Sep 2020

Collectively canalizing Boolean functions

Claus Kadelka, Benjamin Keilty, Reinhard Laubenbacher

2020

arXiv:2008.13741v2 [cs.DM] 5 Oct 2020

Complexity
Volume 2020, Article ID 3687961, 14 pages
https://doi.org/10.1155/2020/3687961

Research Article

The Dynamics of Canalizing Boolean Networks

Elijah Paul,[1] **Gleb Pogudin**,[2,3] **William Qin**,[4] **and Reinhard Laubenbacher**[5]

[1]*California Institute of Technology, Pasadena, CA, USA*
[2]*Courant Institute of Mathematical Sciences, New York University, New York, NY, USA*
[3]*Department of Computer Science, National Research University Higher School of Economics, Moscow, Russia*
[4]*Millburn High School, Millburn, NJ, USA*
[5]*Center for Quantitative Medicine, University of Connecticut Health Center and Jackson Laboratory for Genomic Medicine, Farmington, CT, USA*

Automatica 99 (2019) 167–174

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Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Dynamics of semilattice networks with strongly connected dependency graph☆

Alan Veliz-Cuba[a,*], Reinhard Laubenbacher[b,c]

[a] University of Dayton, Dayton, OH, USA
[b] Center for Quantitative Medicine, University of Connecticut Health Center, USA
[c] Jackson Laboratory for Genomic Medicine, USA

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Advances in Applied Mathematics 30 (2003) 655–678

ADVANCES IN
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Mathematics

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Decomposition and simulation of sequential dynamical systems

Reinhard Laubenbacher [a,*] and Bodo Pareigis [b]

[a] *Virginia Bioinformatics Institute, Blacksburg, VA 24061, USA*
[b] *Mathematisches Institut, Universität München, Theresienstr. 39, D-80333 München, Germany*

Study modularity of nested canalizing
polynomial dynamical systems
over finite fields.

The New York Times

DEADLY GERMS, LOST CURES

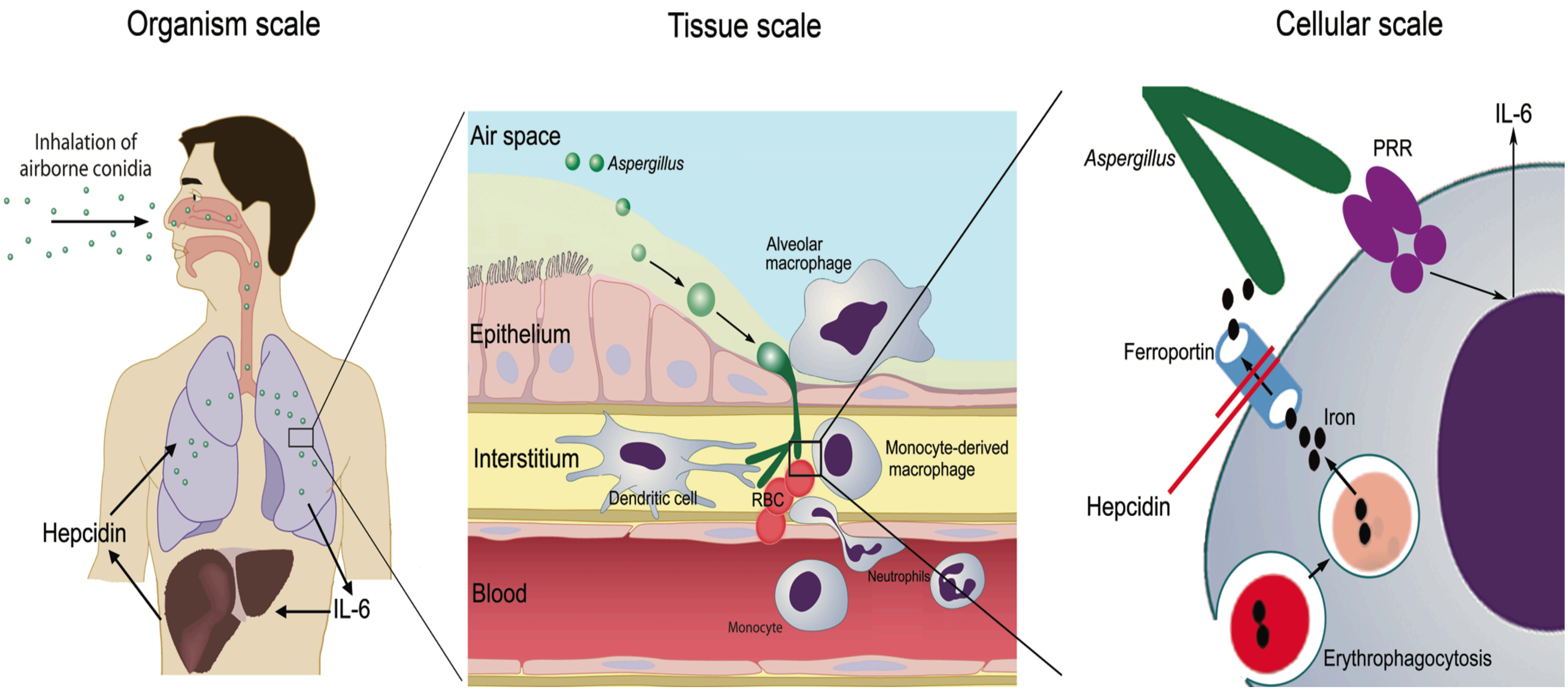
A Mysterious Infection, Spanning the Globe in a Climate of Secrecy

The rise of Candida auris embodies a serious and growing public health threat: drug-resistant germs.

By Matt Richtel and Andrew Jacobs

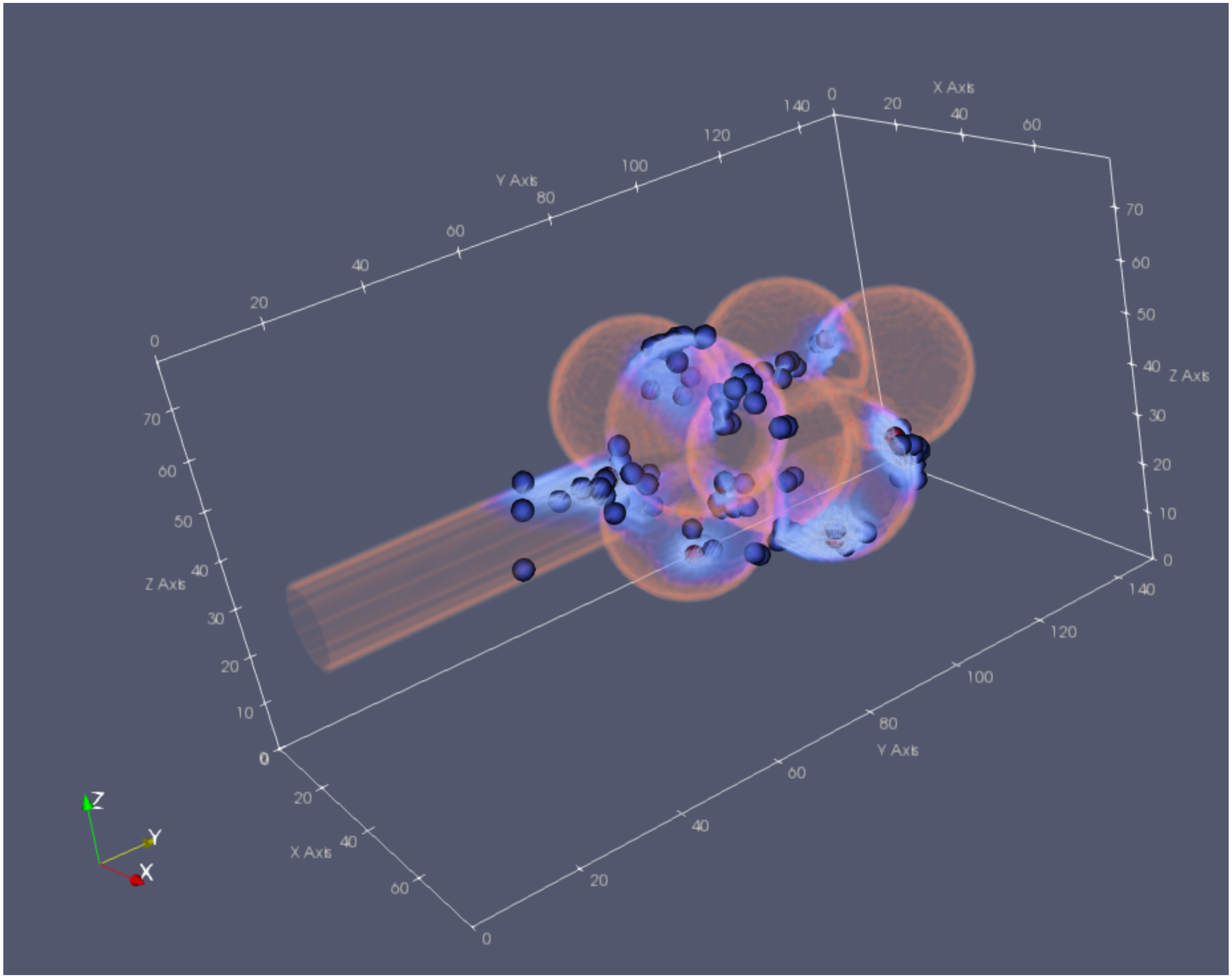
April 6, 2019

Nutritional immunity and fungal infections

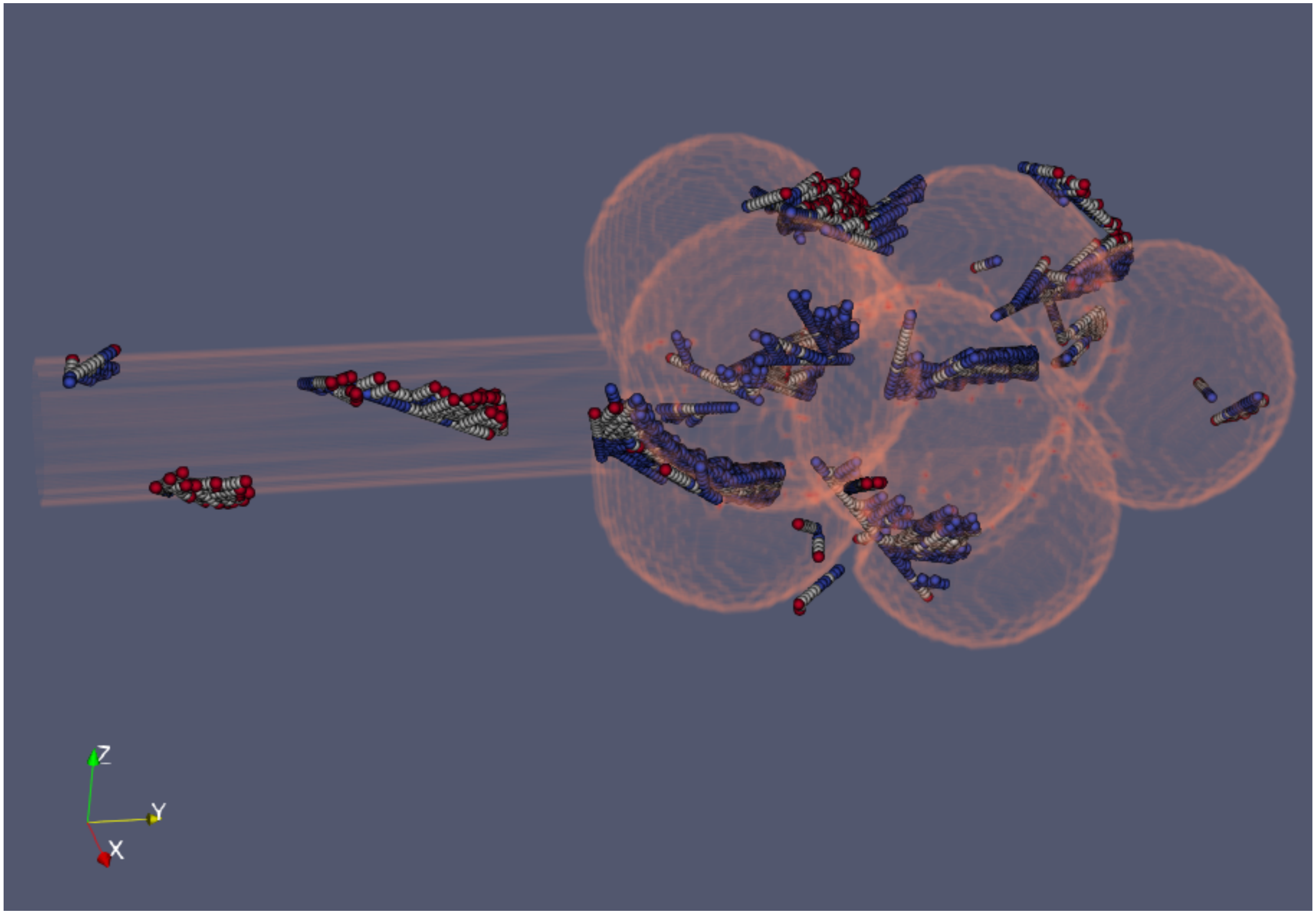


S.J. Park et al., *J. Immunol.*, 2006

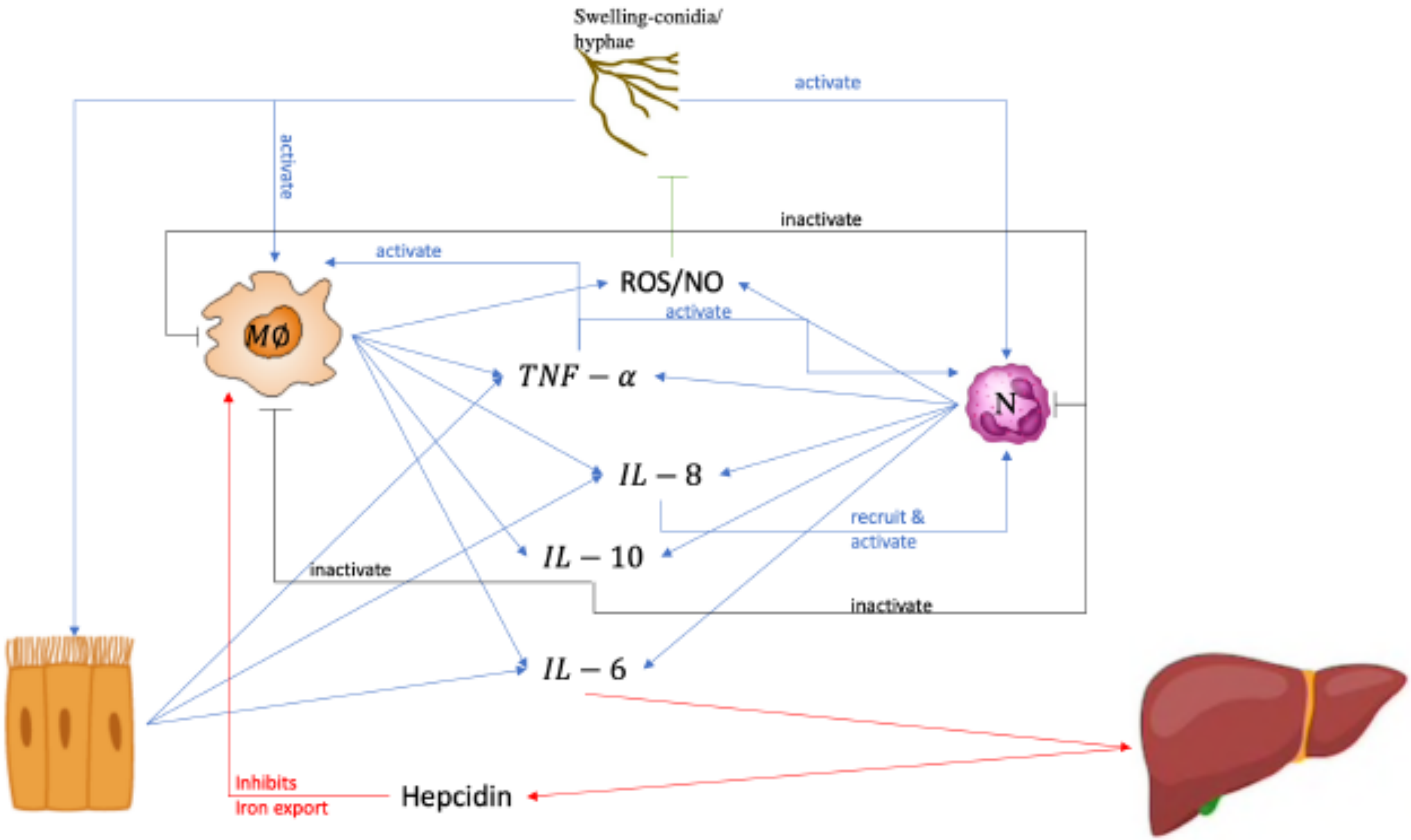
https://smart.servier.com/smart_image/tendon-anatomy/



X Axis
Y Axis
Z Axis
0
20
40
60
80
100
120
140
10
30
50
70
Z
Y
X



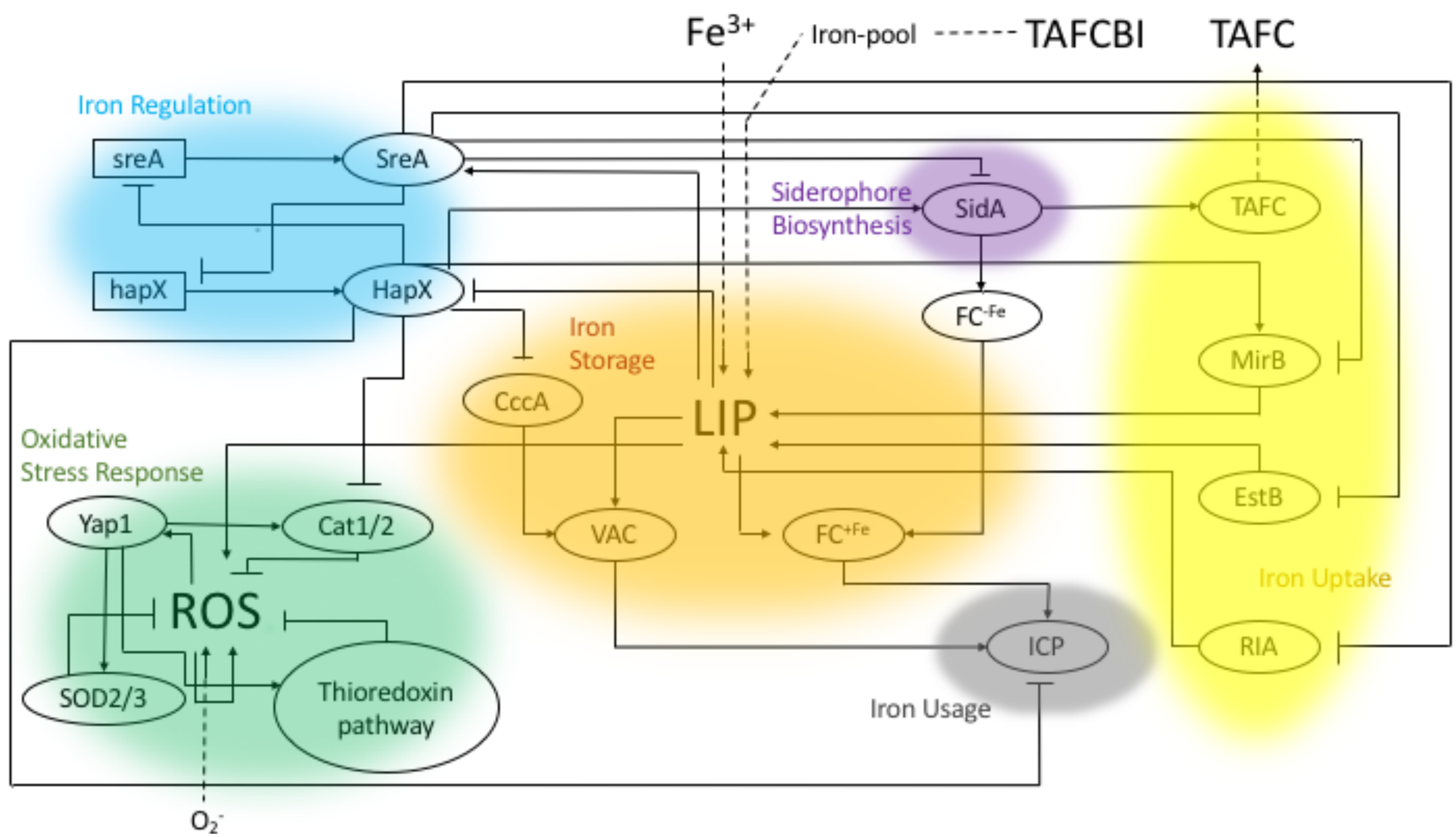
Z
Y
X



Swelling-conidia/
hyphae
activate
activate
inactivate
activate
ROS/NO
activate
MØ
TNF − α
N
IL − 8
recruit &
activate
IL − 10
inactivate
inactivate
IL − 6
Inhibits
Iron export
Hepcidin

Aspergillus fumigatus

Brandon *et al.*,
BMC Sys. Biol. 2015



Some Mathematical Challenges:

- Integration of time scales
- Update schemes for components and overall model
- Model analysis
- Control

<u>The Team:</u>

U Florida:

L. Lab: H. deAssis, L. Fonseca, A. Knapp, E. Mei, B. Shapiro, L. Sordo Vieira

Mehrad Lab: B. Mehrad, N. Yang, Y. Scindia

UConn**:** B. Adhikari, A. Conan, L. Flores, J. Masison, L. Poudel

Kitware Inc.: W. Schroeder, M. Grauer, B. Helba, S. Arikatla, J. Beezley

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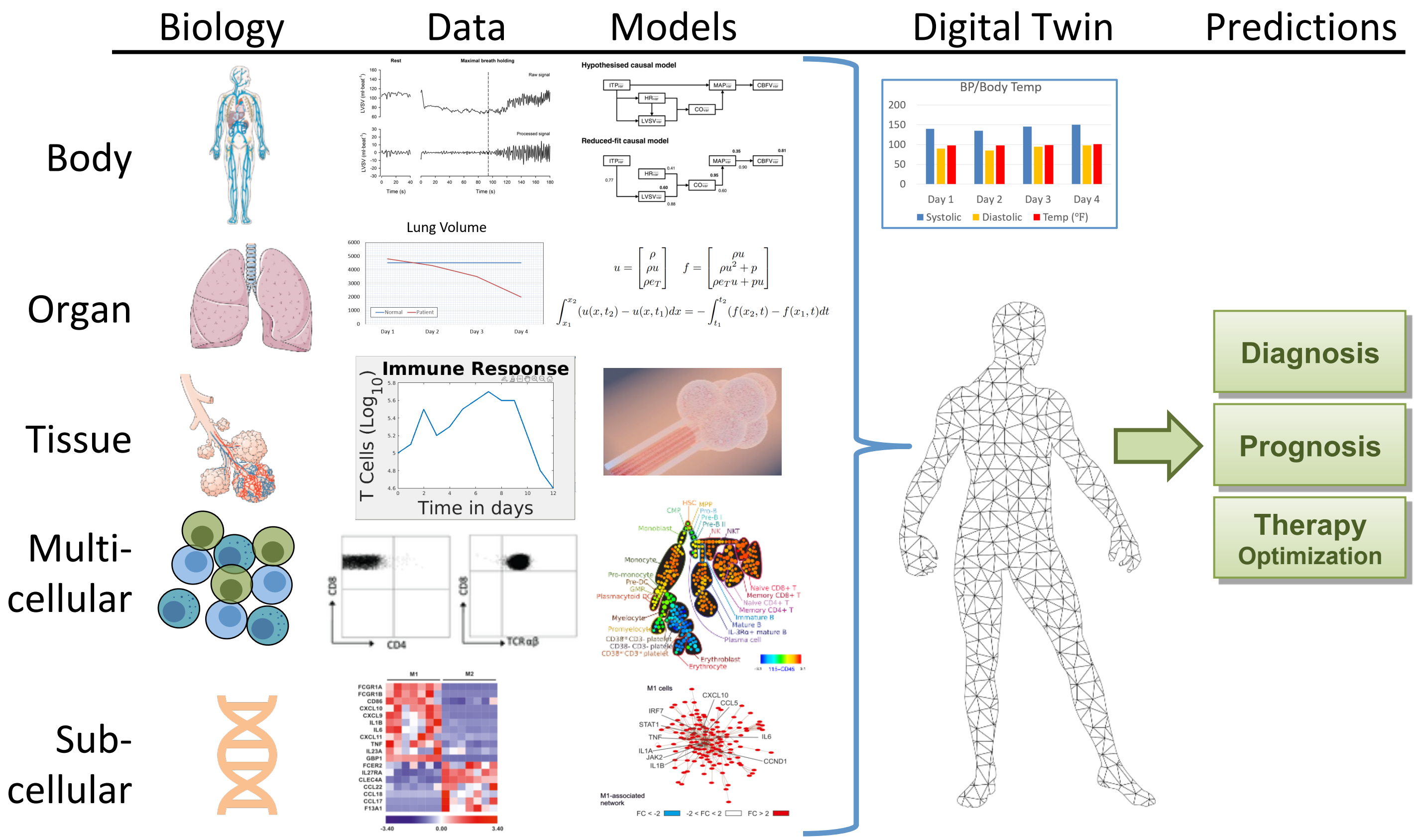
The Medical Digital Twin



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https://smart.servier.com/smart_image/tendon-anatomy/
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