Numerical Analysis – Practise Final Exam

Do four (4) problems – 2 hour time limit

1. Assume $f \in C^2[a,b]$.

   (a) Derive the Trapezoid rule with error

   $$\int_a^b f\,dx = \frac{h}{2}(f(x_0)+f(x_1)) - \frac{h^3}{12}f''(\eta)$$

   (b) Use part (a) to derive the composite Trapezoid rule with error.

2. Show if $g \in C^2[a,b]$ and $g(p)=p$ with $|g'(p)|<1$ then there is an $\epsilon>0$ with $g^n(x)\to p$ for all $x\in[p-\epsilon,p+\epsilon]$.

3. Let $f(x)=\log(x+1)$, $x_0=0, x_1=1, x_2=2$. Find the *total* error bound for the degree two interpolating polynomial $P_2(x)$ with these nodes and f on the interval $[0,2]$, *i.e.* find a K with

   $$\max_{t\in[0,2]} |f(t)-P_2(t)| \le K.$$

4. Find a,b,c so that the quadrature formula

   $$\int_{-1}^{1} f(x)\,dx = af(-1)+bf(0)+cf(1)$$

   has degree of precision as large as possible, and show it is no larger than your answer.

5. With

   $$\phi(w,t) = a[f(w,t)+f(w+ch,t+bh)],$$

   find the values of the parameters a,b,c so that the resulting one-step method

   $$w_0 = \alpha$$
   $$w_{i+1} = w_i + h\phi(w_i,t_i)$$

   has local truncation error $O(h^2)$.