HW5 – Due Friday, March 24, start of class
No electronic submissions, only hard copy
Be sure to justify all your answers completely.

1. *(10 points)* Numerically compute
$$\int_{-1}^{1} \cos(2x + 1/2)\, dx$$
using (a) the composite trapezoid rule, (b) the composite Simpson's rule, and (c) Gaussian Quadrature. In each case first use the error estimates given in class to find how large n needs to be to ensure that the error in your computation is less than 10^{-5}. Then perform the computation for that n and confirm that your answer is within the estimated error.

   For Gaussian Quadrature you can find a list of coefficients and nodes at https://pomax.github.io/bezierinfo/legendre-gauss.html. You can use a computer to compute the error estimates for Gaussian Quadrature.

2. Let $T(a, b)$ and $T(a,(a + b)/2) + T((a + b)/2, b)$ be single and double applications of the Trapezoid rule to $\int_a^b f(x)\, dx$. Derive the approximate relation between
$$|T(a, b) - (T(a,(a + b)/2) + T((a + b)/2, b))|$$
and
$$\left| \int_a^b f(x)\, dx - (T(a,(a + b)/2) + T((a + b)/2, b)) \right|$$
used in adaptive Trapezoid quadrature.

3. Let $h = (b - a)/3$, $x_0 = a, x_1 = a + h$, and $x_2 = b$. Find the degree of precision of the quadrature formula
$$\int_a^b f(x)\, dx \approx \frac{9}{4} h f(x_1) + \frac{3}{4} h f(x_2)$$

4. Derive the error term in Simpson's rule by assuming that
$$\int_a^b f(x)\, dx = \frac{b - a}{6} \left(f(a) + 4f(\frac{a + b}{2}) + f(b) \right) + k f^{(4)}(\eta)$$
and finding k by using the formula for $f(x) = x^4$.