**HW5 – Due Friday, April 7, start of class**
**No electronic submissions, only hard copy**
**Be sure to justify all your answers completely.**

All the problems are concerned with the solution of the IVP on the interval $[a, b]$

$$y' = f(y, t); \quad y(a) = \alpha,$$

with $f$ Lipschitz in $y$ with constant $L$. The solution is denoted $Y(t)$. All methods have step size $h$, $t_0 = a$, and $t_{i+1} = t_i + h$.

1. *(7 points)* Consider the perturbed Euler method

should be f(u_n, t_n) not \phi

$$u_0 = \alpha + \delta_0$$
$$u_{n+1} = u_n + h\phi(u_n, t_n) + \delta_n$$

with the $\delta_n$ representing round-off error. Show that for all $i$,

$$|Y(t_i) - u_i| \leq \frac{1}{L}\left(\frac{hM}{2} + \frac{\delta}{h}\right)\left(e^{L(b-a)} - 1\right) + |\delta_0|e^{L(b-a)}$$

with $\delta = \max|\delta_n|$ and $M = \max|Y''(t)|$, which we assume is finite.

2. *(6 points)* With $\phi(w, t) = af(w + bh, w + ch)$, find the values of the parameters $a, b, c$ so that the resulting one-step method

should be t not w

$$w_0 = \alpha$$
$$w_{i+1} = w_i + h\phi(w_i, t_i)$$

has local truncation error $O(h^2)$.

3. *(12 points)* Consider

$$y' = \frac{2}{t}y + t^2e^t; \quad t \in [1, 2]; \quad y(1) = 0, \tag{1}$$

which has solution $Y(t) = t^2(e^t - e)$

(a) Using the (unperturbed) Euler method error estimate find the value of $h$ required to ensure that the Euler method solution $w_i$ of (1) satisfies $|Y(t_i) - w_i| \leq 0.1$ for all $i$.

(b) Write a program to implement Euler's method and run it with the $h$ value just computed (or more precisely, a value $h$ near it with $1/h = n$, an integer) and confirm that $|Y(t_i) - w_i| \leq 0.1$ for all $i$.

(c) Derive the Taylor method of order 3 for (1).

(d) Write a program to implement this Taylor method and by experimentation, find a value of $h$ which ensures that $|Y(t_i) - v_i| \leq 0.1$, where $v_i$ is the solution computed via your Taylor's method.