HW7 – Due Wednesday, April 26, 5:00 PM
You can turn it in the last day of class, slide it under my office door,
or email it to me as a single pdf file less than 3M.
Be sure to justify all your answers completely.

1. The modified Euler's method is

$$w_0 = \alpha$$
$$w_{n+1} = w_n + \frac{h}{2}\left(f(w_n, t_n) + f(w_n + hf(w_n, t_n), t_{n+1})\right).$$

   Apply this method to the IVP, $y' = \lambda y; y(0) = 1$, with $\lambda < 0$ and find the conditions on λ and h which ensure $w_n \to 0$ as $n \to \infty$.

2. Show that the second column, $R_{k,2}$, of Romberg integration is a composite Simpson's rule.

3. Assume $N(h)$ is the computed approximation for M for each $h > 0$ and

$$M = N(h) + C_1 h + C_2 h^2 + C_3 h^3 + \dots.$$

   Use the values $N(h), N(h/3)$, and $N(h/9)$ to produce a $O(h^3)$ approximation to M.

4. Taylor's formula yields the following.

$$f'(x_0) = \frac{1}{h}(f(x_0 + h) - f(x_0)) - \frac{h}{2}f''(x_0) - \frac{h^2}{6}f'''(x_0) + O(h^3).$$

   Use this with extrapolation to derive an $O(h^3)$ formula for $f'(x_0)$.

5. Let $f(x) = \ln(x)$.

   (a) Find the linear Taylor polynomial $T_1(x)$ of $f(x)$ expanded about $x_0 = 3/2$ and find the maximum error $|T_1(x) - f(x)|$ on $[1, 2]$.

   (b) Find the linear minimax approximation $p_*^{(1)}(x)$ to $f(x)$ on $[1, 2]$ and find the maximum error $|p_*^{(1)}(x) - f(x)|$ on $[1, 2]$.