Numerical Linear Algebra – Fall 2014 – Practise Exam
Do **4** (four) problems

1. (a) Define the spectral radius $\rho(A)$ for a square matrix A.
   (b) Prove that $\rho(A^k) = \rho(A)^k$ for $k > 0$.
   (c) Prove that $\rho(A) \leq \|A\|$ for any induced matrix norm on A.
   (d) For any induced norm, $\|A^k\| \to 0$ as $k \to \infty$ if and only if $\rho(A) < 1$.

2. Prove that a normal triangular matrix is diagonal.

3. Assume $A \in \mathbb{R}^{m,m}$
   (a) Prove that $\langle x, y \rangle_A = x^* A y$ is an inner product on $\mathbb{R}^m$ if and only if A is symmetric and positive definite
   (b) Assume now that A is symmetric and positive definite. If x_* is the solution to $Ax = b$ and $\{p_1, \ldots, p_m\}$ is an orthonormal basis for $\mathbb{R}^m$ with respect to $\langle \ , \ \rangle_A$ and $x_* = \sum c_i p_i$, give a formula for the c_i.

4. (a) If both A and U are in $\mathbb{C}^{m,m}$ and U is unitary, prove that $\|UA\|_2 = \|A\|_2$ and $\|UA\|_F = \|A\|_F$
   (b) Prove that $\|A\|_2 = (\rho(A^* A))^{1/2} = \sigma_1$, where σ_1 is the largest singular values of A.

5. (a) Prove or disprove: If $A = QBQ^*$ with Q unitary, then A and B have the same singular values
   (b) Prove or disprove: If $A = CBC^{-1}$, then A and B have the same singular values