

This will give you an idea of the topics to be covered in the Numerical Linear Algebra PhD exam in January, 2015. They are given approximately in the order they occur in the textbook.

You should also, of course, know all the standard Linear Algebra material contained in an undergraduate course in the subject.

1. Matrix ranks, row and column spaces, null and range spaces
2. Vector norms, induced matrix norms, Fröbenious norm.
3. Inner and outer products, orthogonality
4. Cauchy-Schwarz and Hölder inequalities
5. Complex matrices, adjoints, unitary, Hermitian and positive definite matrices
6. Real matrices, transposes, orthogonal and symmetric matrices
7. Singular value decomposition (SVD)
 - (a) Geometric interpretation
 - (b) Connection to two-norm and Fröbenious-norm and spectrum of A^*A .
 - (c) Low rank approximation theorem
8. Projectors, complementary and orthogonal projectors
9. QR factorization
 - (a) Construction via Gram-Schmidt
 - (b) Construction via Householder reflectors
 - (c) Solving $Ax = b$ via QR factorization.
10. Least squares problems
 - (a) Orthogonal projection and the derivation of the normal equations
 - (b) Solving least squares problems via QR factorization
 - (c) Solving least squares problems via SVD
11. Conditioning and condition number
 - (a) Relative and absolute condition number
 - (b) Condition number of matrix-vector multiplication
 - (c) Condition number of a matrix
12. Floating point arithmetic

13. Stability and backward stability
14. Gaussian elimination, LU factorization and pivoting
15. Cholesky factorization
16. Eigenvalues and eigenvectors – Theory
 - (a) Eigenvalue decomposition
 - (b) Characteristic polynomial, geometric and algebraic multiplicity and defective matrices
 - (c) Similarity transformations
 - (d) Trace and determinant
 - (e) Schur factorization
 - (f) Normal matrices
17. Eigenvalues and eigenvectors – Computation
 - (a) Rayleigh Quotient
 - (b) Power iteration and inverse iteration
 - (c) QR algorithm, with and without shifts
 - (d) Jacobi method
 - (e) Bisection method
 - (f) Divide and conquer method
18. Krylov space methods
 - (a) Arnoldi iteration
 - (b) GMRES
 - (c) Lanczos
 - (d) Conjugate gradient
19. Preconditioning