The protean chromatic polynomial

by

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Let t be a positive integer and let G be a combinatorial graph with vertices V and edges E. A *proper coloring of* G *from a set with* t *colors* is a function $c : V \to \{1, 2, \ldots, t\}$ such that if $uv \in E$ then $c(u) \neq c(v)$, that is, the endpoints of an edge must be colored differently. These are the colorings considered in the famous Four Color Theorem. The *chromatic polynomial* of G, $P(G; t)$, is the number of proper colorings of G from a set with t colors. It turns out that this is a polynomial in t with many amazing properties. One can characterize the degree and coefficients of $P(G; t)$. There are also connections with acyclic orientations, increasing spanning forests, hyperplane arrangements, symmetric functions, and Chern classes in algebraic geometry. This talk will survey some of these results.