Let $t$ be a positive integer and let $G$ be a combinatorial graph with vertices $V$ and edges $E$. A proper coloring of $G$ from a set with $t$ colors is a function $c : V \to \{1, 2, \ldots, t\}$ such that if $uv \in E$ then $c(u) \neq c(v)$, that is, the endpoints of an edge must be colored differently. These are the colorings considered in the famous Four Color Theorem. The chromatic polynomial of $G$, $P(G; t)$, is the number of proper colorings of $G$ from a set with $t$ colors. It turns out that this is a polynomial in $t$ with many amazing properties. One can characterize the degree and coefficients of $P(G; t)$. There are also connections with acyclic orientations, increasing spanning forests, hyperplane arrangements, symmetric functions, and Chern classes in algebraic geometry. This talk will survey some of these results.