Due in hard copy form at the start on class on Friday, Feb. 16. Each answer must include Matlab code (if used), any plots required and the answer to all questions. All figures must have labels.

**Problem A.** Recall the function $\chi_{\pi/4}$ defined on $[-\pi, \pi]$ from the homework given by $\chi_{\pi/4}(t) = 1$ when $|t| < \pi/4$ and $\chi_{\pi/4}(t) = 0$ when $|t| > \pi/4$.

1. Write down the Fourier series of $\chi_{\pi/4}$ (you already did the computation for homework).
2. Plot the first 20 Fourier approximations of $\chi_{\pi/4}$, i.e. $S_n(\chi_{\pi/4})$ for $n = 1, 2, \ldots, 20$
3. Compute the maximum value of $S_n(\chi_{\pi/4})$ for each $n$ and plot it versus $n$.
4. Using your last answer, what is probably the amount of the Gibbs overshoot as $n$ gets large?
5. Write down the Fourier series of $\chi_{\pi/4}$ in orthonormal form.
6. Recall that when $A_k$ and $B_k$ are the Fourier coefficient of a general $f$ in orthonormal form then the squared average error for the $n^{th}$ approximation is

\[
\|E_n\|^2 = \int_{-\pi}^{\pi} (f(t) - S_n(f(t)))^2 dt = \|f\|^2 - (A_0^2 + \sum_{k=1}^{n} A_k^2 + B_k^2).
\]

Use this to make a plot of $\|E_n\|^2$ for $n = 1, \ldots, 20$ for $f(t) = \chi_{\pi/4}(t)$. How big does $n$ have to be so that $\|E_n\|^2$ is less than 5% of $\|\chi_{\pi/4}\|^2$?

**Problem B.**

1. Write down the sine series and cosine series of $f(t) = t$ on $[0, 1]$ you obtained for the homework due on Feb 2.
2. Plot the sine series approximation using the first 5 terms on the interval $[-1, 2]$ (so you will see some of the periodicity).
3. On a separate figure, plot the cosine series approximation using the first 5 terms on the interval $[-1, 2]$ (so you will see some of the periodicity).