Project 1 • Fall 2019 • PROF. BOYLAND

Due in hard copy form at the start on class on Friday, September 27. Each answer must include Matlab code (if used) with the problem number on it, any plots required and the answer to all questions.

Problem A. Recall the function $\chi_{\pi/4}$ defined on $[-\pi, \pi]$ from the homework given by $\chi_{\pi/4}(t) = 1$ when $|t| < \pi/4$ and $\chi_{\pi/4}(t) = 0$ when $|t| > \pi/4$.

1. Write down the Fourier series of $\chi_{\pi/4}$ (you already did the computation for homework).
2. Plot the first 20 Fourier approximations of $\chi_{\pi/4}$, i.e. $S_N(\chi_{\pi/4})$ for $N = 1, 2, \ldots 20$
3. Compute the maximum value of $S_N(\chi_{\pi/4})$ for each N and plot it versus N.
4. Using your last answer, what is probably the amount of the Gibbs overshoot as N gets large?
5. Write down the Fourier series of $\chi_{\pi/4}$ in orthonormal form.
6. Recall that when A_k and B_k are the Fourier coefficient of a general f in orthonormal form then the squared average error for the N^{th} approximation is

$$\|E_N\|^2 = \int_{-\pi}^{\pi} (f(t) - S_n(f)(t))^2 \, dt = \|f\|^2 - (A_0^2 + \sum_{k=1}^{N} A_k^2 + B_k^2).$$

   Use this to make a plot of $\|E_N\|^2$ for $N = 1, \ldots, 20$. How big does N have to be so that $\|E_N\|^2$ is less than 5% of $\|f\|^2$?

Problem B.

1. The cosine series and sine series of $f(t) = t$ on $[0, 1]$ are

$$\frac{1}{2} + \sum_{j=1}^{\infty} \frac{-4}{\pi^2(2j-1)^2} \cos((2j-1)\pi t) \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n\pi} \sin(n\pi t)$$

2. Plot the sine series approximation using the first 8 terms on the interval $[-1, 2]$ so you will see some of the periodicity
3. On a separate figure, plot the cosine series approximation using the first 5 nonzero terms on the interval $[-1, 2]$ so you will see some of the periodicity