Problem A. Recall the function $\chi_{\pi/4}$ defined on $[-\pi, \pi]$ from the homework given by $\chi_{\pi/4}(t) = 1$ when $|t| < \pi/4$ and $\chi_{\pi/4}(t) = 0$ when $|t| > \pi/4$.

1. Write down the Fourier series of $\chi_{\pi/4}$ (you already did the computation for homework).
2. Plot the first 20 Fourier approximations of $\chi_{\pi/4}$, i.e. $S_N(\chi_{\pi/4})$ for $N = 1, 2, \ldots 20$
3. Compute the maximum value of $S_N(\chi_{\pi/4})$ for each $N$ and plot it versus $N$.
4. Using your last answer, what is probably the amount of the Gibbs overshoot as $N$ gets large?
5. Write down the Fourier series of $\chi_{\pi/4}$ in orthonormal form.
6. Recall that when $A_k$ and $B_k$ are the Fourier coefficient of a general $f$ in orthonormal form then the squared average error for the $N^{th}$ approximation is

$$
\|E_N\|^2 = \int_{-\pi}^{\pi} (f(t) - S_n(f)(t))^2 dt = \|f\|^2 - (A_0^2 + \sum_{k=1}^{N} A_k^2 + B_k^2).
$$

Use this to make a plot of $\|E_N\|^2$ for $N = 1, \ldots, 20$. How big does $N$ have to be so that $\|E_N\|^2$ is less than 5% of $\|f\|^2$?

Problem B.

1. The cosine series and sine series of $f(t) = t$ on $[0, 1]$ are

$$
\frac{1}{2} + \sum_{j=1}^{\infty} \frac{-4}{\pi^2(2j-1)^2} \cos((2j-1)\pi t) \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n\pi} \sin(n\pi t)
$$

2. Plot the sine series approximation using the first 8 terms on the interval $[-1, 2]$ so you will see some of the periodicity

3. On a separate figure, plot the cosine series approximation using the first 5 nonzero terms on the interval $[-1, 2]$ so you will see some of the periodicity