

$$A1: f(t) \sim \frac{1}{4} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right) \cos nt$$

A4: between 1.09 and 1.1

$$A5: \sqrt{\frac{\pi}{8}} \left(\frac{1}{\sqrt{2}\pi} \right) + \sum_{n=1}^{\infty} \left(\frac{2}{n\sqrt{\pi}} \right) \left(\sin \frac{n\pi}{4} \right) \left(\frac{\cos nt}{\sqrt{\pi}} \right)$$

$$\|f\|^2 = \int_{-\pi/4}^{\pi/4} f^2 dt = \pi/2$$

$$\text{so } \|E_N\|^2 = \frac{\pi}{2} - \left[\frac{\pi}{8} + \sum_{n=1}^N \frac{4}{n^2\pi} \sin^2\left(\frac{k\pi}{4}\right) \right]$$

$N=9$ almost does it, $N=10$ for sure.

See code files for the rest.