(1) Let $\Phi(x_1, x_2) = 2x_1^3 + x_1 x_2^2 + 5x_1^2 + x_2^2$. Find all critical points and classify them as loc. max, loc min, saddle or no test

(2) Let $A = \begin{bmatrix} 2 & -3 & 0 \\ 4 & -5 & 1 \\ 2 & -1 & 3 \end{bmatrix}$. Find the LU-decomposition of A and use it to solve $A\vec{x} = \begin{bmatrix} 3 \\ 7 \\ 5 \end{bmatrix}$

(3) For a matrix M, define null(M) and show it is a subspace.

(4) Prove or disprove: A and AA have the same null space.

(5) Assume A is $n \times n$ with null$(A) = \{\vec{0}\}$. Let B be the $n \times 3n$ matrix $[A\ A\ A]$. What is null(B)?

(6) Assume $\{\vec{u}_1, \ldots, \vec{u}_n\}$ is an orthonormal basis for $\mathbb{R}^n$ and $\vec{\omega} = \sum_{L=1}^{n} \alpha_L \vec{u}_L$. Show that $\alpha_j = \vec{\omega} \cdot \vec{u}_j$ for all j

(7) Let $A = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$ Solve the differential equation $\frac{d\vec{x}}{dt} = A\vec{x}$ with $\vec{x}(0) = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$. Write your answer in both matrix and vector form.

(8) Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 11 & 6 \\ -15 & -7 \end{bmatrix}$$

(9) Define orthogonal matrix and show that if U is orthogonal, its columns are an orthonormal set.