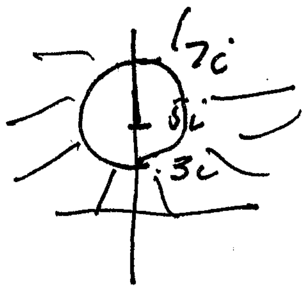
Soln: Review problems exam #1 - Oct. 2017

(1)




(2) $\overline{e^{z}} = \overline{e^{x}e^{iy}} = \overline{e^{x}(\cos y + i\sin y)}$

$= e^{x}(\cos y - i\sin y) = e^{x-iy}$

$= e^{\bar z}$

(3) $8 = 8e^{0+2n\pi i}$ so $8^{1/6} = 8^{1/6}e^{2n\pi i/8}$

$n=0$ principle $8^{1/6}$

$n=1$ $\quad 8^{1/6}e^{\pi i/4} = 8^{1/6}\left(\frac{\sqrt2}{2} + i\frac{\sqrt2}{2}\right)$

$n=2$ $\quad 8^{1/6}e^{\pi i/2} = 8^{1/6}(i)$

$n=3$ $\quad 8^{1/6}e^{3\pi i/4} = 8^{1/6}\left(-\frac{\sqrt2}{2} + i\frac{\sqrt2}{2}\right)$

$n=4$ $\quad 8^{1/6}e^{\pi i} = -8^{1/6}$

$n=5$ $\quad 8^{1/6}e^{5\pi i/4} = 8^{1/6}\left(-\frac{\sqrt2}{2} - i\frac{\sqrt2}{2}\right)$

$n=6$ $\quad 8^{1/6}e^{\frac{3}{2}\pi i} = -8^{1/6}i$

$n=7$ $\quad 8^{1/2}e^{7\pi i/4} = 8^{1/6}\left(\frac{\sqrt2}{2} - i\frac{\sqrt2}{2}\right)$

(4) $f(z) = \frac{z^2}{\bar z} = \frac{z^3}{\bar z z} = \frac{(x+iy)^3}{x^2+y^2}$

$= \frac{x^3 - xy^2}{x^2+y^2} + i\left(\frac{x^2y - y^3}{x^2+y^2}\right)$

(5) Since $|\operatorname{Im} z| \le |z|$

$|\operatorname{Im}(z^3 - 2iz + 1)| \le |z^3 - 2iz + 1|$

$\le |z^3| + |2iz| + 1 = |z|^3 + 2|z| + 1 \le 4$ when $|z| \le 1$

Triangle inequality

(6) $|5+5i| = \sqrt{50}$ so $\mathrm{Log}(5+5i) = \frac{\ln 50}{2} + i\pi/4$

$\mathrm{Arg}(5+5i) = \pi/4$

$\log(5+5i) = \frac{\ln 50}{2} + i\left(\frac{\pi}{4}+2\pi n\right)$

$\left(\frac{\ln 50}{2} = \ln(5\sqrt{2})\right)$

(7) $|2-2i| = \sqrt{8} = 2\sqrt{2}$

$\mathrm{Arg}(2-2i) = -\pi/4$

$\log(2-2i) = \frac{\ln 8}{2} + i\left[-\frac{\pi}{4} + 2\pi n\right]$

$$(2-2i)^i = e^{i\log(2-2i)} = e^{i\left[\frac{\ln 8}{2} + i\left[-\frac{\pi}{4}+2\pi n\right]\right]}$$

$$= \exp\left[\frac{\pi}{4} - 2\pi n\right] \exp\left[i\frac{\ln 8}{2}\right]$$

(8) $\cos(x+iy) = \cos x \cos iy - \sin x \sin iy$

$$= \cos x \frac{e^{iiy}+e^{-iiy}}{2} - \sin x \frac{e^{iiy}-e^{-iiy}}{2i}$$

$$= \cos x \frac{e^{-y}+e^{y}}{2} - \frac{1}{i}\sin x \frac{e^{-y}-e^{y}}{2}$$

$$= \cos x \cosh y - i \sin x \sinh y.$$

(9) $z'(t) = 2ie^{it}$ $\quad \bar{z}(t) = 2e^{-it}$

$$\int_c \bar{z}\, dz = \int_0^{2\pi} (+2e^{-it})(2ie^{it})$$

$$= \int_0^{2\pi} 4i = 8\pi i$$

(10)  $\int_c f\,dz = \int_0^1 t^2 \cdot 1\, dt + \int_1^2 (1+(1-t)i)^2 (-i)\, dt$

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pg 16 #14 $z^2 + \bar{z}^2 = 2$ let $x+iy = z$ becomes:

$(x+iy)^2 + (x-iy)^2 = 2$ or $x^2 + 2ixy - y^2 + x^2 - 2ixy - y^2 = 2$

or $2(x^2-y^2) = 2$ or $x^2 - y^2 = 1$

pg 76, 1b] $u = \cosh x \cos y$ $v = \sinh x \sin y$

$u_x = \sinh x \cos y$ $v_y = \sinh x \cos y$ ✓

$u_y = -\cosh x \sin y$ $v_x = \cosh x \sin y$ ✓

2c) $f(z) = e^y(\cos x + i\sin x)$

so $u = e^y \cos x$ $v = e^y \sin x$

$u_x = -e^y \sin x$ $v_y = e^y \sin x$

$u_y = e^y \cos x$ $v_x = e^y \cos x$

so For CR $u_x = v_y$ since $e^y \neq 0$, $\sin x = -\sin x$

so $x = n\pi$

Also $u_y = -v_x$ so $\cos x = -\cos x$

or $x = n\pi/2$

So no x satisfies both, so CR hold nowhere.

pg 90 #7 With $z = x+iy$, $|\exp(-2z)| = e^{Re(-2z)} = e^{-2x} < 1$ when $x > 0$

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