Review problems - Exam #2 - FALL 17

(1) Compute $\displaystyle\oint_C \frac{e^{2z}}{z\,(z-i)^2}\,dz$ for

(a) $C: z(t) = \frac{1}{4}e^{it} \quad 0 \le t \le 2\pi$

(b) $C: z(t) = \frac{1}{4}e^{it} - i \quad 0 \le t \le 2\pi$

(c) $C: z(t) = \frac{1}{4}e^{it} + i \quad 0 \le t \le 2\pi$

(d) $C: z(t) = 2e^{it}$

(2) evaluate using a single residue "infinity moved to zero"

$\displaystyle\oint_C \frac{(z-2)\,dz}{z(z+3)} \qquad C: z(t) = 5e^{it} \quad 0 \le t \le 2\pi$

(3) Find the Maclaurin series of $f(z) = \dfrac{z}{(1+z^2)^2}$ using differentiation of power series. where does it converge?

(4) Find the Maclaurin series of $f(z) = \mathrm{Log}\,(1+z)$ using integration of another power series. Where does it converge?

(5) Find the Laurent series of $\dfrac{1}{(z-2)(z-3)}$ valid in (a) $0 < |z| < 2$ (b) $2 < |z| < 3$ (c) $|z| > 3$

(6) Use multiplication of power series to find the first 3 non zero terms of the Maclaurin series of

$$f(z) = \frac{\sin z}{z-1}$$

(7) (a) Use division of power series to find the first 3 non zero terms of the Laurent series of $f(z) = \frac{1}{e^z - 1}$ valid in $0 < |z| < 2\pi$

b) Compute $\oint_{|z|=1} \frac{dz}{e^z - 1}$

(8) Compute $\oint_{|z|=3} z^2 \exp\left(\frac{1}{z}\right) dz$

(9) Compute (a) $\oint_{|z-\pi|=1} \frac{\cos z \; \cancel{\sin z}}{(z-\pi)(z+\pi)} dz$ (b) $\oint_{|z|=7} \frac{\cos z \; \cancel{\sin z}}{(z-\pi)(z+\pi)} dz$

(10) compute $\oint_{|z-i|=1/2} \frac{\operatorname{Log} z}{(z^2+1)^2} dz$

(17) Compute $\displaystyle\int_{|z|=4} \cot |z|\, dz$

(18) Compute te real integral using residues

$$\int_0^\infty \frac{dx}{x^4+1}$$ be sure to justify each step fully.

(19) Compute the real integral using residues

$$\int_0^\infty \frac{\cos 5x}{x^2+1}\, dx$$ be sure to justify each step fully