(1) $Df(x_1,x_2) = \begin{bmatrix} 2x_1x_2 & x_1^2 \\ x_2^3 & 3x_1x_2^2 \\ x_2 & x_1 \end{bmatrix}$

$Dg(y_1, y_2, y_3) = \begin{bmatrix} y_2 & y_1 & 0 \\ 0 & y_3 & y_2 \end{bmatrix}$

$Dg(f(x_1,x_2)) = \begin{bmatrix} x_1x_2^3 & x_1^2x_2 & 0 \\ 0 & x_1x_2 & x_1x_2^3 \end{bmatrix}$

$Dh(x_1,x_2) = Dg(f(x_1,x_2)) \cdot Df(x_1,x_2)$ $\longrightarrow$

$= \begin{bmatrix} x_1x_2^3 & x_1^2x_2 & 0 \\ 0 & x_1x_2 & x_1x_2^3 \end{bmatrix} \begin{bmatrix} 2x_1x_2 & x_1^2 \\ x_2^3 & 3x_1x_2^2 \\ x_2 & x_1 \end{bmatrix}$

$$= \begin{bmatrix} 2x_1^2 x_2^4 + x_1^2 x_2^4 & x_1^3 x_2^3 + 3x_1^3 x_2^3 \\ x_1 x_2^4 + x_1 x_2^4 & 3x_1^2 x_2^3 + x_1^2 x_2^3 \end{bmatrix}$$

$$= \begin{bmatrix} 3x_1^2 x_2^4 & 4x_1^3 x_2^3 \\ 2x_1 x_2^4 & 4x_1^2 x_2^3 \end{bmatrix}$$

(2) It is simplest to use the formula for the DFT

$$\hat{X}_j = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k \, \omega^{-kj} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \omega^{-kj}$$

$$= \frac{1}{\sqrt{N}} \left(1 + \omega^{-j} + \omega^{-2j} + \dots + \omega^{-(N-1)j} \right)$$

$$= \frac{1}{\sqrt{N}} \cdot 0 \quad \text{when } j \neq 0$$

using $(1-z)(1+z+\dots+z^{N-1}) = 1-z^N$ and $\omega^{jN} = 1$

$$= \frac{1}{\sqrt{N}} \cdot N \quad \text{when } j = 0$$

So $\text{DFT}(\overrightarrow{\text{one}}) = \begin{bmatrix} \sqrt{N} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

(3) For hand calculations it is ususally easier to find the matrix M

$$u * v = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ -1 \\ 7 \end{bmatrix}$$

(4) (a) $w_1 = \begin{bmatrix}1\\0\end{bmatrix}$ $w_2 = \begin{bmatrix}0\\1\end{bmatrix}$ given basis

$$\|w_1\|^2 = \langle \vec{w}_1, \vec{w}_1 \rangle = [1\ 0]\begin{bmatrix}2&1\\1&1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = [2\ 1]\begin{bmatrix}1\\0\end{bmatrix} = 2$$

$$\text{so } q_1 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1/\sqrt{2}\\0\end{bmatrix}$$

$$\begin{aligned}
V_2 &= w_2 - \langle w_2, q_1 \rangle q_1 \\
&= \begin{bmatrix}0\\1\end{bmatrix} - \left([0\ 1]\begin{bmatrix}2&1\\1&1\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\0\end{bmatrix}\right)\begin{bmatrix}1/\sqrt{2}\\0\end{bmatrix} \\
&= \begin{bmatrix}0\\1\end{bmatrix} - [1\ 1]\begin{bmatrix}1/\sqrt{2}\\0\end{bmatrix}\begin{bmatrix}1/\sqrt{2}\\0\end{bmatrix} \\
&= \begin{matrix}0\\1\end{matrix} - \frac{1}{\sqrt{2}}\begin{bmatrix}1/\sqrt{2}\\0\end{bmatrix} = \begin{bmatrix}-1/2\\1\end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\text{Check } \langle V_2, q_1 \rangle &= [-1/2\ 1]\begin{bmatrix}2&1\\1&1\end{bmatrix}\begin{bmatrix}1/\sqrt{2}\\0\end{bmatrix} \\
&= [0\ 1/2]\begin{bmatrix}1/\sqrt{2}\\0\end{bmatrix} = 0 \ \checkmark
\end{aligned}$$

(4) cont

$$q_2 = \frac{v_2}{\|v_2\|}$$

$$\|v_2\|^2 = \langle v_2, v_2 \rangle = \begin{bmatrix} -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1/2 \end{bmatrix} \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} = 1/2$$

$$\text{so } q_2 = \frac{\begin{bmatrix} -1/2 \\ 1 \end{bmatrix}}{\frac{1}{\sqrt{2}}} = \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2} \end{bmatrix}$$

$$\text{Check } \langle q_2, q_2 \rangle = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \sqrt{2} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \sqrt{2} - \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2} \end{bmatrix}$$

$$= 0 + 2 - \frac{2}{2} = 1 \checkmark$$

4(b) $\vec{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \alpha_1 q_1 + \alpha_2 q_2$

$$\alpha_1 = \langle \vec{v}, q_1 \rangle = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 0 \end{bmatrix} = -1/\sqrt{2}$$

$$\alpha_2 = \langle \vec{v}, \vec{q}_2 \rangle = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2} \end{bmatrix}$$

$$= \sqrt{2}/2$$

so $\begin{bmatrix} -1 \\ 1 \end{bmatrix} = -1/\sqrt{2} \begin{bmatrix} 1/\sqrt{2} \\ 0 \end{bmatrix} + \sqrt{2}/2 \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2} \end{bmatrix}$

$$\underset{\text{check}}{=} \begin{bmatrix} -1/2 \\ 0 \end{bmatrix} + \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \checkmark$$

(5) $$g = \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \\ 0 \\ 1/2 \end{bmatrix} \qquad M = \begin{bmatrix} -1/2 & 1/2 & 0 & 0 & 1/2 \\ 1/2 & -1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & -1/2 & 1/2 \\ 1/2 & 0 & 0 & 1/2 & -1/2 \end{bmatrix}$$

(6) (A) (a)

$$F(x,\eta)=\left[\sigma(\omega_{11}x_1+\omega_{21}x_2+b_1),\ \sigma(\omega_{12}x_1+\omega_{22}x_2+b_2)\right]^T$$

(b) $F(x,\eta)= \gamma_1\,\sigma(\omega_1 x_1+b_1)+\gamma_2\,\sigma(\omega_2 x_1+b_2)$

$+\gamma_3\,\sigma(\omega_3 x_1+b_3)$

(c) $F(x,\eta)=\sigma(\omega_1 x_1+\omega_2 x_2+b)$.

(B) (a) Let $z_1=\omega_{11}x_1+\omega_{21}x_2+b_1$, $z_2=\omega_{12}x_1+\omega_{22}x_2+b_2$

η = w's then b's.

$$DF(x,\eta)=\begin{bmatrix}\sigma'(z_1)x_1 & \sigma'(z_1)x_2 & 0 & 0 & \sigma'(z_1) & 0\\ 0 & 0 & \sigma'(z_2)x_1 & \sigma'(z_2)x_2 & 0 & \sigma'(z_2)\end{bmatrix}$$

(b) η = w's then b's then γ's then B

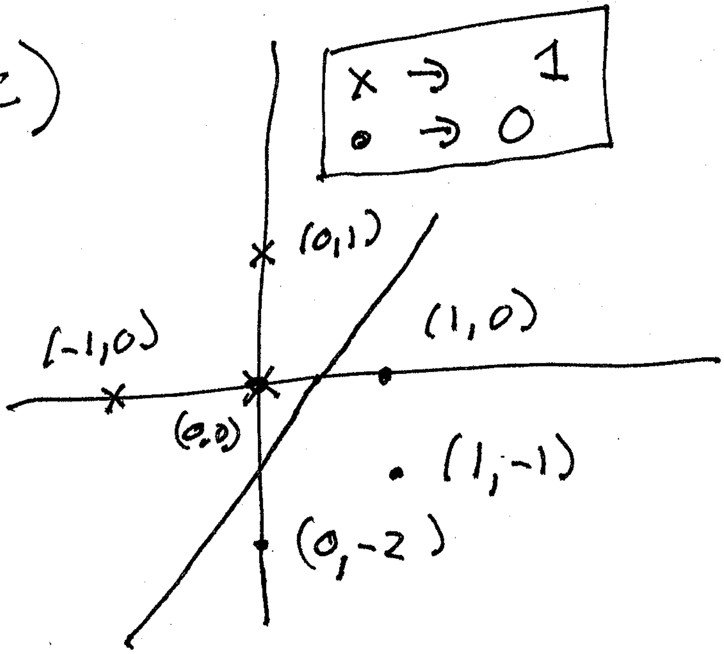
$$DF=\left[\gamma_1\sigma'(z_1)x_1\quad \gamma_2\sigma'(z_2)x_1\quad \gamma_3\sigma'(z_3)x_1 \rightarrow\right.$$

$$\rightarrow \gamma_1\sigma'(z_1)\quad \gamma_2'(z_2)\quad \gamma_3'(z_3)\rightarrow$$

$$\left.\rightarrow \sigma(z_1)\quad \sigma(z_2)\quad \sigma(z_3)\quad 1\right]$$

as single 1×10 vector

6 (c)



one decision line passes through $(0,-1)$ and $(\frac{1}{2},0)$

or $2x_1 - x_2 - 1 = 0$

putting in $(0,0)$ yields -1 and $\sigma_S(-1) = 0$

so we switch the signs or

$w_1 = -2,\ w_2 = 1,\ b = 1$

$F(x,\sigma) = \sigma_S(-2x_1 + x_2 + 1)$

(D) $[\sigma_R(2\cdot 1 + 3(-1) + 3), \sigma_R(1\cdot 1 + 2(-1) + 0)]$

$$= \begin{bmatrix} \sigma_R(2) \\ \sigma_R(-1) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$