1. (10 points) Determine the first four nonzero terms in the Taylor polynomial approximation to the given initial value problem.

\[ y'' = 3y' + xy^2 \quad y(0) = 3; \quad y'(0) = 2 \]

2. (20 points) Find the power series expansion about \( x_0 = 0 \) for the general solution to the given differential equation. Your answer should be in the form of a summation including a general formula for the coefficients.

\[ y'' + y = 0 \]

3. (20 points) Find the first four nonzero terms in the power series expansion about \( x_0 = 0 \) for the general solution to the given differential equation.

\[ y' + (x - 2)y = 0 \]

4. (15 points) For this differential equation

\[ (x^2 - 1)y'' - (x - 1)y' - 3y = 0 \]

(a) Find and classify all singular points as regular or irregular.

(b) Find the indicial equation and exponents at the singularity \( x_0 = -1 \).

(c) For the largest exponent at the singularity \( x_0 = -1 \) write down the form of the corresponding series for the solution.

(d) What is the minimum radius of convergence of the series in (c) and why?

(e) Write down the form of the series for the second linearly independent solution at the singularity \( x_0 = -1 \)

5. (5 points) Write the general solution using Bessel functions

\[ 4x^2 y'' + 4xy' + (4x^2 - 36)y = 0 \]