

Your Name Printed Clearly! \_\_\_\_\_

**Justify all answers! Show all work for partial credit! You will get no credit for just the answer. Note that different problems have different values. No calculators, no notes!**

SAMPLE EXAM 1 • BOYLAND • MAP4305 • SPRING 2019

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1.(10 *points*) Determine the first **four** nonzero terms in the Taylor polynomial approximation to the given initial value problem.

$$y'' = 3y' + xy^2 \quad y(0) = 3; \quad y'(0) = 2$$

2.(20 *points*) Find the power series expansion about  $x_0 = 0$  for the general solution to the given differential equation. Your answer should be in the form of a summation including a general formula for the coefficients.

$$y'' + y = 0$$

3.(20 *points*) Find the first **four** nonzero terms in the power series expansion about  $x_0 = 0$  for the general solution to the given differential equation.

$$y' + (x - 2)y = 0$$

4.(15 *points*) For this differential equation

$$(x^2 - 1)y'' - (x - 1)y' - 3y = 0$$

- (a) Find and classify all singular points as regular or irregular.
- (b) Find the indicial equation and exponents at the singularity  $x_0 = -1$ .
- (c) For the largest exponent at the singularity  $x_0 = -1$  write down the **form** of the corresponding series for the solution.
- (d) What is the minimum radius of convergence of the series in (c) and why?
- (e) Write down the **form** of the series for the second linearly independent solution at the singularity  $x_0 = -1$

5.(5 *points*) Write the general solution using Bessel functions

$$4x^2y'' + 4xy' + (4x^2 - 36)y = 0$$