(A) Find a fundamental matrix for

$$\vec{x}' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \vec{x}$$

(B) Find a fundamental matrix for

$$\vec{x}' = \begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix} \vec{x}$$

Answers on next page

(A) $\begin{bmatrix} e^{3t} & -e^{-t} \\ 2e^{3t} & 2e^{-t} \end{bmatrix}$

(B) If you make different choices of generalized eigenvectors your answer could look different. Char poly $= \lambda(\lambda-5)^2$, $\lambda=0$, $\lambda=5$ double. Here is some work.

$$A-5I = \begin{bmatrix} 0 & -4 & 0 \\ 1 & -5 & 2 \\ 0 & 2 & 0 \end{bmatrix} \qquad (A-5I)^2 = \begin{bmatrix} -4 & 20 & -8 \\ -5 & 25 & -10 \\ 2 & -10 & 4 \end{bmatrix}$$

$(A-5I)^2\vec{u} = 0$ yields just one independent equation $u_1 - 5u_2 + 2u_3 = 0$. My choices

$$\vec{u}_1 = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{x}_1 = e^{5t}\left(\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 & -4 & 0 \\ 1 & -5 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} \right) = e^{5t} \begin{bmatrix} 5-4t \\ 1 \\ 2t \end{bmatrix}$$

$$\vec{x}_2 = e^{5t}\left(\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 & -4 & 0 \\ 1 & -5 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right) = e^{5t} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$\lambda = 0$ has eigen vect $\begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}$ so $\vec{x}_3 = e^{0} \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}$

$$\Phi(t) = \begin{bmatrix} 5e^{5t} - 4te^{5t} & -2e^{5t} & 4 \\ e^{5t} & 0 & 5 \\ 2te^{5t} & e^{5t} & 2 \end{bmatrix}$$