Euler numbers

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Abstract. A permutation $a_1a_2\cdots a_n$ of $1, 2, \ldots, n$ is *alternating* if $a_1 > a_2 < a_3 > a_4 < \cdots$. The number E_n of alternating permutations of $1, 2, \ldots, n$ is called an *Euler number.* We will survey the theory of alternating permutations and Euler numbers, beginning with the famous formula of Désiré André: $\sum_{n\geq 0} E_n \frac{x^n}{n!} = \sec x + \tan x$. Connections will be given to such topics as convex polytopes, tridiagonal matrices, probability theory, and the representation theory of the symmetric group. We will explain how the enumeration of alternating permutations that are also fixed-point-free involutions is related to an asymptotic result in one of Ramanujan's notebooks.