TOPOLOGY EXAM 2.1 • SPRING 2022 • PROF. BOYLAND

1. Give the diagrammatic definition of the free Abelian group on a set S, state how it characterizes the free Abelian group and then prove this characterization.

2. State the Seifert-van Kampen Theorem.

3. Prove that S^2 and S^3 are not homeomorphic.

4. Let $X = \{x \in \mathbb{R}^2 : \|x\| \leq 3\} - \vec{0}$ and $A = \{x \in \mathbb{R}^2 : \|x - 1\| = 1/2\}$. Prove or disprove: A is a retract of X (see figure below).

5. Assuming the usual hypothesis of SVK, show that if $U \cap V$ is simply connected, then

$$\pi_1(X, x_0) \cong \pi_1(U, x_0) * \pi_1(v, x_0).$$

6. (a) Show that the Klein bottle and the torus are not homotopy equivalent.

   (b) Show that the Klein bottle minus a point and the torus minus a point are homotopy equivalent.