

TOPOLOGY EXAM 2.2 • SPRING 2022 • PROF. BOYLAND

1. Consider the Stone-Čech compactification  $\beta\mathbb{N}$  of the natural numbers  $\mathbb{N}$ . Show that the sequence  $a_n = n$  has no convergent subsequences in  $\beta\mathbb{N}$ .
2. Compute the fundamental group of the Klein bottle minus  $n$  points with  $n > 0$ .
3. Let  $X = \{0, 1\}^{\mathbb{N}}$  with the product topology. For two sequences say  $\underline{s} \sim \underline{t}$  if and only if the sequences agree after some point, i.e. there is an  $N > 0$  so that  $s_i = t_i$  for all  $i > N$ . Check that this is an equivalence relation and show that  $X/\sim$  with the quotient topology is compact.
4. Using the Seifert-Van Kampen Theorem compute the fundamental group of the space pictured below.
5. ~~(a) Define a chain complex of Abelian groups  
(b) Define the homology of a chain complex of Abelian groups  
(c) Define a chain map between chain complexes of Abelian groups  
(d) Show that a chain map induces a homomorphism of homology for each  $n$   
(e) Show that for each  $n$ , this assignment of a chain complex and chain maps to Abelian groups and homomorphisms is a functor~~
6.  $X$  is a simplicial complex and the Smith Normal Form of the boundary maps  $\partial_1$  and  $\partial_2$  are given below. What is  $H_1(X, \mathbb{Z})$ ?

**not on qual**

#5

$$\text{SNF}(J_1) = \begin{bmatrix} 1 & & & 0 & 0 & 0 & 0 \\ & 1 & & 1 & 1 & 1 & 1 \\ & & 1 & & & & \\ & & & 3 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 \\ & & & & & 0 & 0 \\ & & & & & & 0 \end{bmatrix}$$

$$\text{SNF}(J_2) = \begin{bmatrix} 1 & & & & & & 0 \\ & 1 & & & & & \vdots \\ & & 4 & & & & \vdots \\ & & & 0 & & & \vdots \\ & & & & 0 & & \vdots \\ & & & & & 0 & \vdots \\ & & & & & & 0 \\ & & & & & & 0 \\ & & & & & & 0 \end{bmatrix}$$

All missing entries are zero.

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#4

