Read all these instructions before starting. After you have read them, turn the page and start timing your exam.

1. You have 1.5 hours to do the exam. This does not include the time that it might take you to scan your soln and upload them. You may also write your solutions directly on a tablet.
2. Exams must be uploaded to Assignments by 5:00 PM, Thursday, December 16 (FL time)
3. The exam is open book and notes, but only those for this course.
4. Set aside 1.5 hours where you can work undisturbed. During this time you cannot talk to anyone or consult any web resources other than those for this course.
5. The questions sheets are a list of the questions and there is no place to put your answers, so write your answers on separate pieces of paper. You can print the question sheet, or look at it on-screen, but do not include them in your scanned soln.
6. Keep in mind that I have to grade these, so please write neatly, organize your answers, and have your solutions in the same order as the questions.
7. Give complete answers. In your proofs you may use any result that we have proved in class or any homework or review problem.
8. You are bound to all the stated conditions for the exam by the UF Honor code.
9. Your scanned solutions must contain the statement:

   "I have spent at most 1.5 hours working the exam and while I was taking the exam I have consulted no one nor nor any book nor any web resources other than the web pages for this course"

   Then your signature and UF ID number.

TOPOLOGY EXAM 2 • FALL 2021 • PROF. BOYLAND

1. Show that the Tietze Extension Theorem implies Urysohn's Lemma.
2. Assume (X, d) is a metric space such that every ball $B_\epsilon(x)$ has compact closure. Show the space is complete.
3. Let A_λ be a collection of connected sets for $\lambda \in \Lambda$ some index set. Assume that A is also connected and $A \cap A_\lambda \neq \emptyset$ for all λ. Show that $A \cup (\cup_{\lambda \in \Lambda} A_\lambda)$ is connected.
4. Assume $f : [0, 1] \to [0, 1]$ is continuous. Show that there is a point x_0 with $f(x_0) = x_0$. Is the same true for a continuous $f : [0, 1) \to [0, 1)$? Be sure to justify your answer completely.
5. (a) Show that a closed subspace of a Lindelöf space in Lindelöf.

   (b) Show by example that a closed subspace of a separable space need not be separable.
6. Suppose that (X, d) is a metric space and A and B are nonempty, disjoint, compact subsets. Define
$$d(A, B) = \inf\{d(a, b) : a \in A \text{ and } b \in B\}.$$
   Prove that there exist points $c \in A$ and $k \in B$ such that $d(A, B) = d(c, k) > 0$.

Don't forget your signed statement