TOPOLOGY HW 2.3 • SPRING 2022 • PROF. BOYLAND

This HW uses material from Lectures 11-16.

1. Give the diagrammatic definition of the free group on a set S (as in lecture 2.15), state how it characterizes the free group and then prove this characterization.
2. State how version 1 of the SVC Theorem in Lecture 2.16 characterizes $\pi_1(X, x_0)$ and then prove this characterization.
3. Prove that these spaces are *not* homeomorphic
   (a) $\mathbb{R}^1$ and $\mathbb{R}^n$ when $n > 1$
   (b) $\mathbb{R}^2$ and $\mathbb{R}^n$ when $n > 2$
4. Compute the fundamental groups of:
   (a) $S^1 \times D^2$ (D^2 is the unit closed two-dimensional disk)
   (b) $S^1 \times S^2$
5. Compute the fundamental groups of the spaces on the next page. You have to give some justification in each case such as SV Theorem, homotopy equivalence, product or some combination of these.

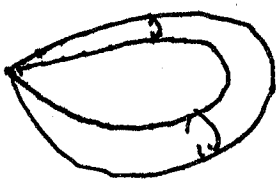
(4)  Two spheres joined at a point

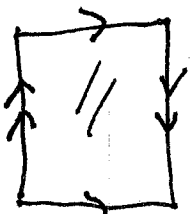
(5)  Two tori joined at a point

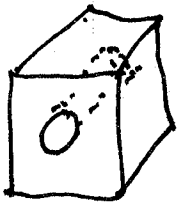
(6)  sphere with an arc attached

(7)  Torus with an arc attached

(8)  disk with a hollow tub attached.

(9)  pinched torus

(10)  Square with edges identified as drawn.

(11)  solid cube with a hole drilled out