TOPOLOGY HW 5 • FALL 2021 • PROF. BOYLAND

This HW uses material from Lectures 23-28.

1. Assume A and B are compact subsets of the Hausdorff spcae X. Prove or disprove:
   (a) $A \cap B$ is compact
   (b) $A \cup B$ is compact
2. Show that a compact subset of a metric space is bounded.
3. In the lower limit topology $\mathbb{R}_\ell$ is $[0,1]$ compact, is it limit point compact? Justify your answer.
4. If A and B are disjoint compact subsets of Hausdorff space X, then there exist open sets U and V with $A \subset U$ and $B \subset V$ and $U \cap V = \emptyset$.
5. In $[0,1]^{\mathbb{Z}_+}$ with the uniform topology find an infinite subset with no limit point and thus with this topology $[0,1]^{\mathbb{Z}_+}$ is not compact.
6. Do problem 6 on page 178 of Munkres on the Cantor middle third set.
7. Do problem 7a (just part a) on page 182 of Munkres on a version of the contraction mapping theorem.
8. Let $X = \mathbb{R} \times \{0,1\}$ be given the product topology with $\mathbb{R}$ having the standard topology and $\{0,1\}$ the indiscrete topology where the only open sets are $\{0,1\}$ and $\emptyset$.
   (a) Show that X is limit point compact.
   (b) Show that projection onto the first coordinate $\mathbb{R}$ is continuous and onto.
   (c) Show that $\mathbb{R}$ is not limit point compact.
9. Let $f : X \to Y$ be continuous and bijective and X is limit point compact, show that Y is limit point compact. (The previous problem shows this isn't true without the injectivity assumption).