TOPOLOGY HW 6 • FALL 2021 • PROF. BOYLAND

This HW uses material from Lectures 29 - 34.

1. If X is regular, show that every pair of disjoint points have neighborhoods that have disjoint closures.
2. If X is normal, show that every pair of disjoint closed sets have neighborhoods that have disjoint closures.
3. Show that every compact metric space is second countable.
4. Show that the countable product (with the product topology) of separable spaces is separable (watch out, the countable product of countable sets is not countable).
5. Show that the real line $\mathbb{R}$ with the usual topology can be written as the countable union $\cup_{n \in \mathbb{Z}_+} A_n$ where each A_n has empty interior (the A_n don't need to be disjoint and they can't be closed by the Baire Theorem).
6. A space is called *locally Baire* if every point has an open neighborhood that is a Baire space. Prove that locally Baire implies Baire.
7. Assume that (X, d) is metric. Show that
   (a) Lindelöf implies second countable
   (b) Separable implies second countable
8. Show that (X, d) is complete if and only if any nested family of closed sets A_n with $\mathrm{diam}(A_n) \to 0$ has $\cap A_n \neq \emptyset$