

TOPOLOGY HW 7 • FALL 2021 • PROF. BOYLAND

This HW uses material from Lectures 33-37. It is worth 15 points, half of a usual HW.

1. Show that a closed subspace of a normal space is normal.
2. Show that a regular Lindelöf space is normal
3. Show that a connected normal space having more than one point is uncountable (Hint: Urysohn's Lemma).
4. Give a direct proof of Urysohn's Lemma for a metric space using the function

$$f(x) = \frac{d(x, A)}{d(x, A) + d(x, B)}$$

5. Show that a compact HD space is metrizable if and only if it is second countable.
6. Let X have the property that every pair of disjoint closed sets can be separated by a continuous function. Show that X is normal.