1. Let $Y$ be the set of all sequences in $\{0,1\}^{\mathbb{Z}^+}$ that are eventually zero. So $x \in Y$ if and only if there exists an $N$ so that $x_i = 0$ for $i \geq N$. Show that $Y$ is countable.

2. Construct a homeomorphism from $\{0,1\}^{\mathbb{Z}^+}$ to a proper subset of itself where $\{0,1\}^{\mathbb{Z}^+}$ is given the product topology.

3. Prove or disprove.
   
   (a) $\overline{A \cap B} = \overline{A} \cap \overline{B}$
   
   (b) $\overline{A \cup B} = \overline{A} \cup \overline{B}$
   
   (c) $\overline{A - B} = \overline{A} - \overline{B}$
   
   (d) $\text{Int}(A \times B) = \text{Int} A \times \text{Int} B$
   
   (e) $\text{Int}(A \cup B) = \text{Int} A \cup \text{Int} B$
   
   (f) $\text{Int}(A - B) = \text{Int} A - \text{Int} B$

4. Give $\mathbb{R}$ the finite complement topology.
   
   (a) To which point or points does the sequence $x_n = 1/n$ converge.
   
   (b) What is $\{2\}$.

5. Assume $A \subset X$ and $f : A \to Y$ is continuous with $Y$ Hausdorff. If there are continuous $g : \overline{A} \to Y$ and $h : \overline{A} \to Y$ that both extend $f$ i.e. $f(a) = g(a) = h(a)$ for all $a \in A$ show that $f = g$. This says that continuous functions extend uniquely to the closure.

6. Let $B \subset \mathbb{R}^{\mathbb{Z}^+}$ be defined as
   
   $B = \prod_{i \in \mathbb{Z}^+} (-1,1) = (-1,1) \times (-1,1) \times (-1,1) \times \ldots$
   
   (a) Is $B$ open in the box topology?
   
   (b) Is $B$ open in the uniform topology?
   
   (c) Is $B$ open in the product topology?

7. If $(X,d)$ is a metric space and $A \subset X$ for $x \in X$ define
   
   $d(x, A) = \inf\{d(x, a) : a \in A\}$.
   
   (a) Show that the function $f : X \to [0, \infty)$ defined by $f(x) = d(x, A)$ is continuous
   
   (b) Show that $x \in \overline{A}$ if and only if $f(x) = 0$.

8. If $(X,d)$ is a metric space show that for any $x$ and $\epsilon > 0$, 
   
   $B_{\epsilon}(x) \subset \{y \in X : d(y, x) \leq \epsilon\}$.
9. If \((X, \mathcal{T}_X)\) is a topological space and \(f : X \to Y\) is a surjective function, is
\[
\{f(U) : U \in \mathcal{T}_X\}
\]
a topology on \(Y\)? Prove your answer.

10. Assume \(A \subset X\).
   (a) Show that \(\text{Bd}(A)\) is a closed set
   (b) Prove that if \(\text{Bd}(A) = \emptyset\) then \(A\) is both open and closed.