This exam covers material from Lectures 19-34. Also review HW 2.4-2.6.

1. (a) If X and Y are completely regular and homeomorphic, show that their Stone-Čech compactifications are homeopmorphic.

   (b) Show that $\beta\mathbb{N}$ is homeomorphic to $\beta\mathbb{Z}$.

2. Give $\{0,1\}$ the discrete topology. Show that $\{0,1\}^{\mathbb{Z}}$ with the product topology is compact.

3. ~~Define a category and a functor and show that the composition of two functors is a functor.~~

4. Compute the fundamental group of the two-dimensional torus minus $n > 0$ distinct points.

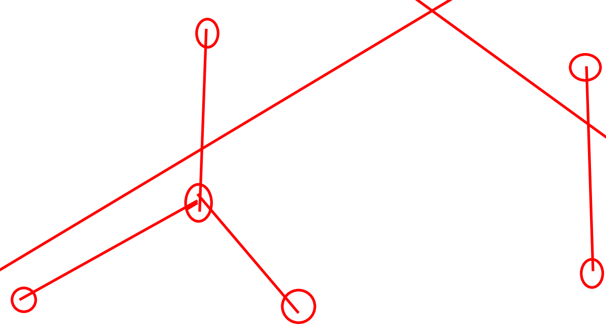
5. Let $\sim$ and $\bowtie$ be equivalence relations on X and Y respectively. $f : X \to Y$ is a continuous function such that $x \sim x'$ implies $f(x) \bowtie f(x')$, Show that f induces a continuous function $X/\!\sim \to X/\!\bowtie$.

6. State the hypothesis on $X, U, V, U \cap V, x_0$ in the SVK Theorem. Show that if U and V are simply connected, then so is X.

7. Let $Z = \{0,1\}$, $X = \{a,b,c\}$, and $Y = \{\alpha, \beta\}$. $f : Z \to X$ is $f(0) = a, f(1) = b$ and $g : Z \to y$ is $g(0) = \alpha = g(1)$. Describe the pushout in the category of Sets.

8. Define a simplex, a simplicial complex, and simplicial homology with $\mathbb{Z}$ coefficients and compute the homology of the simplex pictured below.

9. Recall that a poset gives rise to a simplicial complex. Which poset gives rise to the standard k-simplex?



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