HW1 – Due Friday, September 16, start of class

(1) In a metric space (X, d) show that a set U is open if and only if its complement $U^c = X \setminus U$ is closed.

(2) Define three dilations of the real line.

$$f_1(x) = x/5, f_2(x) = x/5 + 2/5, f_3(x) = x/5 + 4/5.$$

Let $K_0 = [0, 1]$ and for $n = 1, 2, 3, \ldots$ let

$$K_{n+1} = f_1(K_n) \cup f_2(K_n) \cup f_3(K_n)$$

and

$$K = \bigcap_{n=0}^{\infty} K_n.$$

(a) Draw a picture of K_1 and K_2.

(b) Show that K contains no intervals.

(c) If $.a_1a_2a_3\ldots$ with $a_i \in \{0, 1, 2, 3, 4\}$ is the base 5 expansion of a point $x \in [0, 1]$, show that K is the collection of all points $x \in [0, 1]$ so that x has a base 5 representation $x = .a_1a_2a_3\ldots$ with each $a_i \neq 1, 3$.

(d) Show that K is uncountable.

(3) In a metric space (X, d) if $x_n \to a$ and $x_n \to b$, show that $a = b$

(4) For the fractal construction below with Q_0 the unit square, give formulas for the four dilations f_i so that

$$Q_{n+1} = f_1(Q_n) \cup f_2(Q_n) \cup f_3(Q_n) \cup f_4(Q_n).$$