# Bisection is Linearly Convergent

Assume that $r$ is a root to $f(x)=0$ in the interval $[a,b]$. Letting $a_1=a$ and $b_1=b$, the bisection algorithm produces a sequence of approximations of the form $x_n=\frac{a_n+b_n}{2}$. For $n\geq 1, b_n-a_n=\frac{1}{2^{n-1}}(b-a)$. Then $|x_n-r|\leq\frac{b-a}{2^n}$ and $|x_{n+1}-r|\leq\frac{b-a}{2^{n+1}}$. Hence

$$\lim_{n\to\infty}\left|\frac{x_{n+1}-r}{x_n-r}\right|=\lim_{n\to\infty}\left|\frac{(b-a)/2^{n+1}}{(b-a)/2^n}\right|=\frac{1}{2}$$

which shows that bisection is linearly convergent.