Fixed-Point Iteration is Linearly Convergent

Assume that $x_{i+1} = g(x_i)$ converges to r, where g' is continuous and satisfies $g'(r) \neq 0$. By the Mean Value Theorem, there exists c_n between x_n and r such that

$$x_{n+1} - r = g(x_n) - g(r) = g'(c_n)(x_n - r)$$

Since $\{x_n\}$ converges to r, $\{c_n\}$ also converges to r. Hence,

$$\lim_{n\to\infty} \frac{x_{n+1} - r}{x_n - r} = \lim_{n\to\infty} g'(c_n) = g'(r)$$

$$\Rightarrow \lim_{n\to\infty} \left| \frac{x_{n+1} - r}{x_n - r} \right| = |g'(r)|$$

which shows that fixed-point iteration is linearly convergent.