Newton's Method is Quadratically Convergent

Assume that r is a root to $f(x)=0$ and $f'(x)\neq 0$. The sequence $\{x_i\}$ of Newton's algorithm is $x_{i+1}=x_i-\dfrac{f(x_i)}{f'(x_i)}$. From Taylor's Theorem, we have

$$f(r)=P_1(r)+R_1(r)=f(x_i)+f'(x_i)(r-x_i)+\frac{f''(c)}{2!}(r-x_i)^2$$

where c is between r and x_i. Since $f(r)=0$,

$$\begin{aligned}
&0=f(x_i)+f'(x_i)(r-x_i)+\frac{f''(c)}{2!}(r-x_i)^2\\
&\Rightarrow \frac{-f(x_i)}{f'(x_i)}=(r-x_i)+\frac{f''(c)}{f'(x_i)}\frac{(r-x_i)^2}{2}\\
&\Rightarrow x_i-\frac{f(x_i)}{f'(x_i)}-r=x_{i+1}-r=\frac{f''(c)}{f'(x_i)}\frac{(r-x_i)^2}{2}\\
&\Rightarrow \frac{|x_{i+1}-r|}{|x_i-r|^2}=\left|\frac{f''(c)}{2f'(x_i)}\right|=\lambda
\end{aligned}$$

which shows that Newton's method is quadratically convergent.