Name: ______________________________________

MAS 4105 Test 1 Spring 2014

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1. (12 points) Consider the following five vectors from R^4.

$$u_1=(1,2,3,4),u_2=(2,5,7,9),u_3=(1,1,2,3),u_4=(0,1,2,-1),u_5=(0,0,-2,4)$$

Hint: The matrices A and B are row equivalent.

$$A=\begin{pmatrix}1&2&1&0&0\\2&5&1&1&0\\3&7&2&2&-2\\4&9&3&-1&4\end{pmatrix}\quad B=\begin{pmatrix}1&0&3&0&-4\\0&1&-1&0&2\\0&0&0&1&-2\\0&0&0&0&0\end{pmatrix}$$

(a) Do these five vectors span R^4?

(b) Is the set $\{u_1,u_2,u_3\}$ linearly independent?

(c) Is the set $\{u_1,u_2,u_4\}$ linearly independent?

(d) Is u_5 in the span of $S=\{u_1,u_2,u_3,u_4\}$?

2. (9 points) Let W_1 and W_2 be subspaces of a vector space V. Prove that the sum W_1+W_2 is a subspace of V.
3. (9 points) Let S be a linearly independent subset of a vector space V. Let $v\in V, v\notin S$. Prove that if $S\cup\{v\}$ is linearly dependent, then $v\in\operatorname{Span}(S)$.
4. (10 points) Find the dimensions of the following vector spaces.

(a) The subspace $W=\{(x,y):x=y\}\subseteq R^2$.

(b) The subspace of $M_{4\times 4}(R)$ consisting of all 4 by 4 symmetric matrices.

(c) The subspace of $M_{6\times 6}(R)$ consisting of the diagonal matrices.

(d) The subspace W of $P_4(R)$ given by $W=\{f : f(2)=0\}$.

(e) The subspace of R^3 consisting of all solutions to the system of equations

$$x_1+x_3=0$$
$$x_2-x_3=0$$