Name: ______________________________________

MAS 4105 Test 2 Spring 2014

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1. (10 points) Let $T: M_{2\times 2}(R) \to R$ be the linear transformation

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + c.$$

(a) Find a basis for $N(T)$.

(b) Find a basis for $R(T)$.

(c) Let β and γ be the standard ordered bases for $M_{2\times 2}(R)$ and R, respectively. Compute $[T]_\beta^\gamma$.

2. (10 points) Let V and W be vector spaces, and $T: V \to W$ a linear transformation. (a) Define $N(T)$, and (b) Prove that $N(T)$ is a subspace of V.

3. (10 points) Let V and W be vector spaces and $T: V \to W$ a one-to-one linear transformation. Prove that T carries linearly independent subsets of V onto linearly independent subsets of W.

4. (10 points) Let $V, W,$ and Z be vector spaces, and let $T: V \to W$ and $U: W \to Z$ be linear transformations.

(a) Prove or disprove: If UT is one-to-one, then T is one-to-one.

(b) Prove or disprove: If UT is onto, then T is onto.