

Name: _____

MAS 4105

Test 3

Spring 2014

1. (12 points) Let $T : P_1(\mathbb{R}) \rightarrow P_1(\mathbb{R})$ be the linear transformation $T(a + bx) = (b - a) - bx$. Let $\beta = \{1, x\}$ and $\gamma = \{1 + x, 1 - x\}$ be ordered bases.

(a) Calculate $[T]_{\beta}^{\beta}$.

(b) Calculate $Q = [I]_{\gamma}^{\beta}$.

(c) Calculate $[T]_{\gamma}^{\gamma}$.

(d) Calculate T^{-1} . That is, find $T^{-1}(a + bx)$.

2. (10 points) Let V and W be vector spaces, and let $T : V \rightarrow W$ an invertible linear transformation. Prove that the inverse of T is linear.

3. (9 points) Let A be an invertible matrix. Prove that A^t is invertible.

4. (9 points) A square matrix Q is called **orthogonal** if $QQ^t = I$. Prove that if Q is orthogonal, then $\det(Q) = \pm 1$.