

Name: _____

MAS 4105

Test 3

Spring 2015

1. (10 points) Let $\beta = \{(1,0), (0,1)\}$ and $\gamma = \{(0,1), (-1,3)\}$ be ordered bases for \mathbb{R}^2 . Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation $T(x, y) = (2x + y, -3x - 2y)$.

(a) Find the change of coordinate matrix $Q = [I]_{\gamma}^{\beta}$.

(b) Calculate Q^{-1} .

(c) Find $[T]_{\beta}^{\beta}$.

(d) Find $[T]_{\gamma}^{\gamma}$.

(e) Calculate $T^{-1}(x, y)$.

2. (9 points) Prove Theorem 3.9: Let K be the solution set of a system of linear equations $Ax = b$, and let K_H be the solution set of the corresponding homogeneous system $Ax = 0$. Let p be a particular solution to $Ax = b$. Prove that $K = \{p\} + K_H$.

3. (7 points) Let A and B be $n \times n$ matrices. Prove or disprove: If A and B are similar, then $\det(A) = \det(B)$.

4. (7 points) Let A and B be $n \times n$ matrices. Prove or disprove: If $AB = 0$, and $A \neq 0$, then B is invertible.

5. (7 points) Let A and B be $n \times n$ matrices. Prove or disprove:
 $\det(A + B) = \det(A) + \det(B)$