Name:______________________________

MTG 3212 Test 1 Spring 2013

Show all your work on these four sheets. You may use the back. 35 points total.

1. (15 points) For each of the examples below, determine which incidence axioms hold, and which parallel postulates hold. Circle the ones that hold. Hint: Look at the last page of the test.

(a) The Three-Point Plane: *Point* means one of the three symbols A, B, C, the *lines* are the sets $\{A, B\}, \{A, C\}, \{B, C\}$, and *lie on* means "is an element of".

A1 A2 A3 P1 P2 P3

(b) The Square Geometry: *Point* means one of the four vertices of a square, *line* means one of the sides of the square, and *lie on* means that the vertex is an endpoint of the side.

A1 A2 A3 P1 P2 P3

(c) The Klein Disk: *Point* means a point in the Cartesian plane such that the point lies inside the unit circle. *Line* is the part of a Euclidean line that lies inside the circle, and *lie on* has its usual Euclidean meaning.

A1 A2 A3 P1 P2 P3

(d) The Sphere: *Point* means a point on the surface of the sphere $x^2+y^2+z^2 = 1$. The *lines* are great circles on the sphere, and *lie on* means "is an element of".

A1 A2 A3 P1 P2 P3

(e) The Three-Point Line: *Point* means one of the three symbols A, B, C, the *line* is the set $\{A, B, C\}$, and *lie on* means "is an element of".

A1 A2 A3 P1 P2 P3

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2. (10 points) Use the proof technique shown in the textbook and class to prove that $\sqrt{5}$ is irrational.

3. (10 points) Consider the Cartesian plane with the usual Euclidean metric:

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Let ℓ be a nonvertical line with equation $y = mx + b$. Define $f : \ell \to \mathbf{R}$ by $f(x, y) = x\sqrt{1 + m^2}$. Show that f is a one-to-one correspondence satisfying $PQ = |f(P) - f(Q)|$.

The Three Incidence Axioms.

A1. For every pair of distinct points P and Q there exists exactly one line ℓ such that both P and Q lie on ℓ.

A2. For every line ℓ there exists at least two distinct points P and Q such that both P and Q lie on ℓ.

A3. There exist three points that do not all lie on any one line.

The Three Parallel Postulates.

P1. For every line ℓ and for every point P that does not lie on ℓ, there is exactly one line m such that P lies on m and $m \parallel \ell$.

P2. For every line ℓ and for every point P that does not lie on ℓ, there is no line m such that P lies on m and $m \parallel \ell$.

P3. For every line ℓ and for every point P that does not lie on ℓ, there are at least two lines m and n such that P lies on both m and n, and both m and n are parallel to ℓ.