1. Show that for any RV $X$, $V(X) \geq 0$. (You can assume $X$ to be discrete, but this result holds in general.) Hence or otherwise show that $E(X^2) \geq E^2(X)$.

2. (WMS, Problem 3.43.) Many utility companies promote energy conservation by offering discount rates to consumers who keep their energy usage below certain established subsidy standards. A recent EPA report notes that 70% of the island residents of Puerto Rico have reduced their electricity usage sufficiently to qualify for discounted rates. If five residential subscribers are randomly selected from San Juan, Puerto Rico, find the probability of each of the following events:

   (a) All five qualify for the favorable rates.

   (b) At least four qualify for the favorable rates.

3. (WMS, Problem 3.41.) A multiple-choice examination has 15 questions, each with five possible answers, only one of which is correct. Suppose that one of the students who takes the examination answers each of the questions with an independent random guess. What is the probability that he answers at least ten questions correctly?

4. Suppose a balanced coin is tossed 20 times. Define the two random variables $X$ = total number of heads, and $Y$ = total number of tails in the 20 tosses.

   (a) Show that $X$ and $Y$ have the same distribution, i.e., show that the two random variables $X$ and $Y$ have the same support and the same PMF.

   (b) Find the probability that $X$ and $Y$ are the same, i.e., $P(X = Y)$. [**Note:** This example shows that two different random variables can have the same distributions. In fact, as we can see in this case, $P(X = Y)$ is pretty small, although they have the same distribution.]

   (c) Show that the distribution of $X$ is symmetric about $20/2 = 10$, i.e., show that for any $j \in \mathbb{R}$, $P(X = 10 + j) = P(X = 10 - j)$.

5. (WMS, Problem 3.77.) If $Y$ has a geometric distribution with success probability $p$, show that $P(Y = \text{an odd integer}) = \frac{p}{1-q^2}$. [**Hint:** Observe that here we are not interested in a particular odd integer, rather, we want the probability that $Y$ is *any* odd integer.]

6. (WMS, Problem 3.72.) Given that we have already tossed a balanced coin ten times and obtained zero heads, what is the probability that we must toss it at least two more times to obtain the first head? [**Hint:** Save a lot of work by using the memoryless property of the Geometric distribution.]

7. (WMS, Problem 3.76.) Of a population of consumers, 60% are reputed to prefer a particular brand, A, of toothpaste. If a group of randomly selected consumers is interviewed, what is the probability that exactly five people have to be interviewed to encounter the first consumer who prefers brand A? At least five people?

8. Let $X \sim \text{Geo}(p)$. Find the DF $F_X$ of $X$, and comment on the discontinuity points of $F_X$.