1. (WMS, Problem 4.140.) Identify the distributions of the random variables with the following moment-generating functions:
   (a) $M(t) = (1 - 4t)^{-2}$.
   (b) $M(t) = 1/(1 - 3.2t)$.
   (c) $M(t) = e^{-5t+6t^2}$.

2. The life (in hours) of an electronic tube manufactured by a certain process is normally distributed with mean 160 and variance σ^2. What is the maximum allowable value for σ if the life of a tube is to have a probability 0.8 of being between 120 and 200 hours? (Leave the answer in terms of the inverse the standard normal CDF Φ.)

3. Suppose that X has the beta distribution with parameters α and β, and let r and s be given positive integers. Determine the value of $E[X^r(1 - X)^s]$.

4. (WMS, Problem 4.129.) During an eight-hour shift, the proportion of time Y that a sheet-metal stamping machine is down for maintenance or repairs has a beta distribution with $\alpha = 1$ and $\beta = 2$. That is, $f(y) = 2(1 - y), 0 \leq y \leq 1$. The cost (in hundreds of dollars) of this downtime, due to lost production and cost of maintenance and repair, is given by $C = 10 + 20Y + 4Y^2$. Find the mean of C.

5. Let X have a beta distribution with parameters $\alpha > 1$ and $\beta > 1$. Recall that the mode of (the distribution of) X is the maximizer of the density of X. Find the mode of X.

6. (WMS, Problem 4.145.) A random variable Y has the density function

$$f(y) = \begin{cases} e^y, & y < 0 \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Find $E(e^{3Y/2})$.
   (b) Find the moment-generating function for Y.
   (c) Find $V(Y)$.

7. A RV X is said to have the *Cauchy distribution* with location parameter $\mu \in \mathbb{R}$ and scale parameter $\sigma > 0$ if X has density

$$f(x) = \frac{\sigma}{\pi} \frac{1}{\sigma^2 + (x - \mu)^2}. \; -\infty < x < \infty$$

   (a) Find the CDF F of X. [**Hint:** Use the fact that $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan(\frac{x}{a}) + C$ where C is an arbitrary constant of integration.]
   (b) For $0 < p < 1$, find the p-th quantile ϕ_p of (the distribution of) X. [**Hint:** Note that F is continuous and strictly increasing on $\mathbb{R}$.]
   (c) Show that μ is the median of X.

8. (WMS, Problem 5.3.) Of nine executives in a business firm, four are married, three have never married, and two are divorced. Three of the executives are to be selected for promotion. Let Y_1 denote the number of married executives and Y_2 denote the number of never-married executives among the three selected for promotion. Assuming that the three are randomly selected from the nine available, find the joint probability function of Y_1 and Y_2.