1. (WMS, Problem 6.15.) Let Y have a distribution function given by

$$F(y) = \begin{cases} 0 & y < 0, \\ 1 - e^{-y^2}, & y \geq 0. \end{cases}$$

Find a transformation $G(U)$ such that, if U has a uniform distribution on the interval (0,1), $G(U)$ has the same distribution as Y.

2. (WMS, Problem 6.20) Let the random variable Y possess a uniform distribution on the interval (0,1). Derive the

   (a) distribution of the random variable $W_1 = Y^2$.

   (b) distribution of the random variable $W_2 = \sqrt{Y}$.

3. (WMS, Problem 6.33.) The proportion of impurities in certain ore samples is a random variable Y with a density function given by

$$f(y) = \begin{cases} (3/2)y^2 + y, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

The dollar value of such samples is $U = 5 - (Y/2)$. Find the probability density function for U.

4. Suppose $X \sim \text{Bin}(n, p)$. Find the distribution of $Y = n - X$. For which value of p will Y and X have the same distribution?

5. Let $Z \sim N(0, 1)$. Use the method of transformation to show that $X = \mu + \sigma Z$, for $\sigma > 0$ has $N(\mu, \sigma^2)$ distribution.

6. (WMS, Problem 6.88.) Suppose that the length of time Y it takes a worker to complete a certain task has the probability density function given by

$$f(y) = \begin{cases} e^{-(y-\theta)}, & y > \theta, \\ 0, & \text{elsewhere,} \end{cases}$$

where θ is a positive constant that represents the minimum time until task completion. Let $Y_1, \cdots, Y_n$ denote a random sample of completion times from this distribution. Find

   (a) the density function for $Y_{(1)} = \min(Y_1, Y_2, ..., Y_n)$.

   (b) $E(Y_{(1)})$.

7. Let $Y_1, Y_2, \cdots, Y_n$ be independent, uniformly distributed random variables on the interval $[0, \theta]$. Find the PDF of $Y(n) = \max\{Y_1, Y_2, ..., Y_n\}$.

8. Suppose $X_1, \cdots, X_n$ denote a random sample from $\text{Exp}(\beta)$ distribution. Find the distribution of the sample mean $\overline{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$.