1. Elections are being held for two offices, with Democratic and Republican candidates (and no other parties or independent candidates) for both offices. Define the events

$$A = \{\text{the first office is won by a Democratic candidate}\}$$
$$B = \{\text{the second office is won by a Democratic candidate}\}$$

Describe the following events using unions, intersections and/or complements of A and B.

   (a) The second office is won by a Republican.

   (b) Both offices are won by Democrats.

   (c) At least one of the offices is won by a Democrat.

   (d) Both offices are won by Republicans.

   (e) At least one of the offices is won by a Republican.

2. (WMS, Problem 2.11.) A sample space consists of five simple events, E_1, E_2, E_3, E_4, and E_5.

   (a) If $P(E_1) = P(E_2) = 0.15, P(E_3) = 0.4$, and $P(E_4) = 2P(E_5)$, find the probabilities of E_4 and E_5.

   (b) If $P(E_1) = 3P(E_2) = 0.3$, find the probabilities of the remaining simple events if you know that the remaining simple events are equally probable.

3. (WMS, Problem 2.18.) Suppose two balanced coins are tossed and the upper faces are observed.

   (a) List the sample points for this experiment.

   (b) Assign a reasonable probability to each sample point. (Are the sample points equally likely?)

   (c) Let A denote the event that exactly one head is observed and B the event that at least one head is observed. List the sample points in both A and B.

   (d) From your answer to part (c), find $P(A)$, $P(B)$, $P(A \cap B)$, $P(A \cup B)$ and $P(\bar{A} \cup B)$.

4. (WMS, Problem 2.5.) Let $\mathcal{S}$ denote the sample space (universe) and A, B denote two arbitrary events (sets). Use the identities $A = A \cap \mathcal{S}$ and $\mathcal{S} = B \cup \bar{B}$, and the distributive law to prove that

   (a) $A = (A \cap B) \cup (A \cap \bar{B})$.

   (b) If $B \subseteq A$ then $A = B \cup (A \cap \bar{B})$.

   (c) Further, show that $(A \cap B)$ and $(A \cap \bar{B})$ are mutually exclusive, and therefore, A is the union of two mutually exclusive sets, $(A \cap B)$ and $(A \cap \bar{B})$. [This is called partitioning of A with respect to B.]

   (d) Also show that B and $(A \cap \bar{B})$ are mutually exclusive and if $B \subseteq A$, A is the union of two mutually exclusive sets, B and $(A \cap \bar{B})$.

5. (WMS, Problem 2.21-2.24.) Let A and B be two events.

(a) Use the results derived in problem 4 and the Axioms from class to prove that $P(A) = P(A \cap B) + P(A \cap \bar{B})$.

(b) Suppose $B \subseteq A$. Use the result proved in part (a) to show that $P(A) = P(B) + P(A \cap \bar{B})$.

(c) Finally, using non-negativity of probability show that $P(B) \leq P(A)$, if $B \subseteq A$. [That is, if A contains B then the probability of A cannot be smaller than that of B.]

6. (WMS, Problem 2.28.) Four equally qualified people apply for two identical positions in a company. One and only one applicant is a member of a minority group. The positions are filled by choosing two of the applicants at random.

   (a) List the possible outcomes for this experiment.

   (b) Assign reasonable probabilities to the sample points.

   (c) Find the probability that the applicant from the minority group is selected for a position.