STA 4321/5325 **Homework 2** **January 13, 2017**

1. If k people are seated in a random manner in a *circle* containing n chairs ($n > k$), what is the probability that the people will occupy k adjacent chairs in the circle? [**Hint**: Note that a circle has no beginning or end. So, for $k = 2$, along with the usual $(n-1)$ pairs we have when the chairs are on a row (as discussed in class), in this case we have one more pair, namely, $(n, 1)$. What happens for general k (i.e., $k \geq 2$)? Drawing a circle and plotting n points on it might help you better visualize the problem.] [**Answer:** $\frac{(n-k)!\,k!}{(n-1)!}$.]

2. (WMS, Problem 2.53.) Five firms, $F_1, F_2, ..., F_5$, each offer bids on three separate contracts, C_1, C_2, and C_3. Any one firm will be awarded at most one contract. The contracts are quite different, so an assignment of C_1 to F_1, say, is to be distinguished from an assignment of C_2 to F_1.

   (a) How many sample points are there altogether in this experiment involving assignment of contracts to the firms? (No need to list them all.)

   (b) Under the assumption of equally likely sample points, find the probability that F_3 is awarded a contract. [**Answer:** 3/5.]

3. (WMS, Problem 2.64.) A balanced die is tossed six times, and the number on the uppermost face is recorded each time. What is the probability that the numbers recorded are 1, 2, 3, 4, 5, and 6 in any order (i.e., each of the six possible numbers appears)? [**Answer:** 5/324.]

4. An exam consists of 10 multiple choice questions, each with two possible answers - one correct and one incorrect. A student is trying to answer the questions by guessing at random.

   (a) What is the probability that the student gets at most two correct answers? [**Answer:** $\frac{56}{1024}$.]

   (b) What is the probability that she gets at least eight correct answers? [**Answer:** $\frac{56}{1024}$.]

5. Suppose that 100 mathematics students are divided into five classes, each containing 20 students, and that awards are to be given to 10 of these students. If each student is equally likely to receive an award, what is the probability that exactly two students in each class will receive awards? [**Answer:** $(190)^5\ 10!\,90!/100!$.]

6. Each of 50 families has two children. A group of 50 children is chosen at random.

   (a) What is the probability that a given family is represented? [**Answer:** $\frac{149}{198}$.]

   (b) What is the probability that all families are represented? [**Answer:** $\frac{2^{50}.\,50!\,50!}{100!}$.]

7. A deck of 52 cards contains four aces. If the cards are shuffled and distributed in a random manner to four players so that each player receives 13 cards, what is the probability that all four aces will be received by the same player? [**Answer:** $\frac{10.11.12}{49.50.51}$.]

8. (WMS, Problem 2.68.) Show that, for any integer $n \geq 1$,

   (a) $\binom{n}{0} = 1$. Interpret this result.

   (b) $\binom{n}{n} = 1$. Interpret this result.

   (c) $\binom{n}{r} = \binom{n}{n-r}$, for any $r = 0, 1, \cdots, n$. Interpret this result.

(d) $\sum_{i=0}^{n} \binom{n}{i} = 2^n$. [**Hint**: Consider the binomial expansion of $(x+y)^n$ with $x = y = 1$.]

9. (WMS, Problem 2.69.) Prove that, for all positive integers n and k ($n \geq k$), $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

10. A company has 20 new jobs for which it has recruited 20 employees. There are 6 jobs in City 1, 4 jobs in City 2, 5 jobs in City 3 and 5 jobs in City 4. Out of the 20 employees, 4 are friends. Assuming that the company gives no preference to any person in assigning jobs, find the probability that all 4 friends land in the same city.