1. Let A and B be two independent events.
   (a) Recall from Homework exercise 1.5 that $A = (A \cap B) \cup (A \cap \overline{B})$, with $(A \cap B)$ and $(A \cap \overline{B})$ mutually exclusive and $P(A) = P(A \cap B) + P(A \cap \overline{B})$. Using these results or otherwise, prove that A and $\overline{B}$ are independent. [**Note:** Because A and B are arbitrary, we can reverse the roles of A and B to prove that $\overline{A}$ and B are independent.]
   (b) Prove that $\overline{A}$ and $\overline{B}$ are independent. [**Hint:** Start with $P(\overline{A \cup B}) = 1 - P(A \cup B)$, and use additive law.] Thus, from part (a) and (b), it follows that if A, B are independent, then so are $(\overline{A}, B)$, $(A, \overline{B})$ and $(\overline{A}, \overline{B})$.

2. Suppose that two balls are selected at random, without replacement, from a box containing r red balls and b blue balls. Find the probability that the first ball is red and the second is blue. [**Answer:** $P(A \cap B) = \frac{rb}{(r+b)(r+b-1)}$]

3. (WMS, Problem 2.94.) A smoke detector system uses two devices, A and B. If smoke is present, the probability that it will be detected by device A is 0.95; by device B, 0.90; and by both devices, 0.88.
   (a) If smoke is present, find the probability that the smoke will be detected by either device A or B or both devices. [**Answer:** 0.97.]
   (b) Find the probability that the smoke will be undetected. [**Answer:** 0.03.]

4. Three dice are rolled. If no two show the same face, what is the probability that at least one die shows an ace (i.e., a single dot)? [**Hint:** Consider the complementary event of getting no ace.] [**Answer:** 1/2.]

5. (WMS, Problem 2.99.) Suppose that A and B are independent events such that the probability that neither occurs is a and the probability of B is b. Show that $P(A) = \frac{1-b-a}{1-b}$.

6. (*Boole's* inequality for two events.) Let A and B be two (arbitrary) events. Prove that $P(A \cup B) \leq P(A) + P(B)$.

7. (WMS, Problem 2.111.) An advertising agency notices that approximately 1 in 50 potential buyers of a product sees a given magazine ad, and 1 in 5 sees a corresponding ad on television. One in 100 sees both. One in 3 actually purchases the product after seeing the ad, 1 in 10 without seeing it. What is the probability that a randomly selected potential customer will purchase the product? [**Hint:** Define the events: $A = \{\text{buyer sees the magazine ad}\}$, $B = \{\text{buyer sees the TV ad}\}$ and $C = \{\text{buyer purchases the product}\}$.] [**Answer:** $\frac{1}{3} \times 0.21 + 0.10 \times 0.79 = 0.149$.]